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**Favoritism in public
procurement auctions: model
of endogenous entry**

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Abstract

Governments of different countries try to lower the entry cost in public procurement in order to decrease public spending. The purpose of this paper is to examine how the entry cost influences favoritism and procurement prices in the corrupt environment. We adapt the model of selective entry and find that lower entry cost always reduces the contract price paid by the benevolent procurer, but at the same time may make favoritism more stable. Thus the entry cost does not affect the contract price paid by the corrupt procurer or increase it. We illustrate this result using case study on gasoline procurement in Russia where the entry cost of companies was decreased by e-procurement reform. This allows us to examine how changes in entry costs influence competition of companies and procurement prices in auctions.

JEL Classification: H57, D73

Key words: public procurement; endogenous entry; favoritism; e-auctions

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1. Introduction

To take part in a public auction each company has to incur non-zero entry cost: fill out an application form, submit technical documentation, provide an auctioneer with the financial guarantee and other necessary information and finally make a bid. This high entry cost may prevent companies from entry in public procurement leading, potentially, to public waste. It is still an open question, however, whether lowering entry cost decreases public spending.

Models of auctions with endogenous entry address this question. They assume that the number of companies in an auction is not exogenous, but depends on a number of factors, such as the entry cost and the reserve price inside the model. One can identify two key models of endogenous entry: the model of selective entry by Samuelson (1985) and the model of non-selective entry by Levin and Smith (1994). Although they differently model the moment when a company learns its private costs of executing the public contract, these models agree upon an idea that more bidders do not always decrease the contract price. The authors of subsequent theoretical and empirical studies (Hubbard, Paarsch, 2009; Krasnokutskaya, Seim, 2011; Kjerstad, Vagstad, 2000; Li, Zheng, 2009) adapt one of these models to various environmental settings or examine their differences and applicability to data analysis. Yet to the best of our knowledge, no paper explores how corruption affects entry and bidding of companies. As corruption is wide-spread in public procurement of different countries (Bandiera, Prat, Valletti, 2009; Boehm, Olaya, 2006; Søreide, 2002), we believe it is important to fill this gap in the literature.

The purpose of this paper is to examine how the entry cost affects the sustainability of corruption and the contract price paid by the procurer. We focus on such type of corruption as favoritism, which means that the public procurer can extract a bribe only from one company (a potential favorite company) in exchange for a public contract award. We are mainly interested in the market for a simple homogeneous product where each company knows its production costs before it enters the auction. The model of selective entry Samuelson (1985) is more appropriate for analyzing this situation, therefore we choose it for adaptation to corrupt environment. In our model procurer sets contract terms that are either manipulative, that means only one company meets them, or non-manipulative, that means each company in the market meets them. A simple example of gasoline procurement illustrates the idea of manipulating contract terms. For instance, the procurer knows such unique characteristics of a particular company as the distance to his office, the number of gasoline stations in the district and special payment methods. If the procurer includes these characteristics into contract terms, only one company can enter the auction. The corrupt procurer can use this manipulation in exchange for a bribe, while the benevolent procurer can use it to guarantee the delivery of the gasoline.

The main conclusions of the model are as follows. Reducing the entry cost leads to lower contract price paid by the benevolent procurer, while the contract price paid by the corrupt procurer may change in any direction depending on the initial size of the entry cost and the magnitude of its decline. For instance, if the initial entry cost is at a medium level and then drops dramatically, the contract price paid by the corrupt procurer increases and becomes equal to the maximum reserve price in the auction. A negative link between the entry cost and the bribe is the driving force for this change. The lower the entry cost is, the more favoritism is attractive to the public procurer. Thus, an exogenous change in the entry cost, such as the reform of public procurement, may lead to opposite changes in prices paid by benevolent and corrupt procurers.

The rest of the paper is organized as follows. In the Section 2 we present main assumptions and description of the game and in the Section 3 – the solution to the basic model. Then we analyze the impact of the entry cost on contract prices in the Section 4 and provide ideas for extensions of the basic model in the Section 5. In the Section 6 we analyze two cases in Russian public procurement of gasoline. The Section 7 concludes.

2. Main assumptions and the description of the game

2.1 General description of the game

A public procurer wants to buy one unit of homogeneous indivisible product that gives him value $v, v = 1$. The market consists of $n \geq 2$ companies⁴ capable to deliver the product. Each company $i, i \in [1; n]$ incurs costs of two types: production costs c_i and the entry cost k . Production costs are directly related to the contract execution, while the entry cost includes all preparatory costs of a company necessary to enter the auction. For example, each company has to prepare and submit the necessary documentation, guarantee its financial and participate in the auction. Production costs are independently and identically distributed on the interval $[0; 1]$ with c.d.f. $F(c), F(0) = 0, F(1) = 1$ and density function $f(c)$. For the sake of simplicity we assume that $c_i \sim U[0; 1]$. Entry costs of all companies are identical, $k_i = k, 1 \geq k \geq 0$.

To purchase the product the procurer organizes the reverse first-price sealed-bid auction, according to the standard rules. The reserve price in the auction equals $r, r < 1 + k$, and is set exogenously⁵. We assume that the entry cost is big enough to prevent a company with high production costs from entering the auction and winning it at the reserve price, $k > r - 1$. All participating companies simultaneously and secretly make bids $P_i(c_i), i = 1, \dots, n$. Bids higher than the reserve price are not accepted, $P_i(c_i) < r$. Then the procurer discloses all bids and announces the winner that is a company that made a lowest bid. The winner must execute the

⁴ Hereinafter we refer to a procurer as “he” and to a company as “she”.

⁵ Further we will relax this assumption.

public contract at a price equal to its bid. In equilibrium, a bid of each company positively depends on its production costs and exceeds them: $P_i'(c_i) > 0, P_i(c_i) > c_i$ (Krishna 2002, 16–19). If none of the companies enter, the auction is void and all players receive zero payoffs. This assumption is accordance with the current procurement practice and was used in the previous literature (e.g. Auriol, 2006).

As in the Samuelson's model, each company learns the exact value of her the entry cost before she decides to enter the auction. Hence, the certain threshold level of production costs c^* exists: a company enters the auction, only if her production costs are under this level. Information about the production costs of the company is her private information, while the distribution of production costs, the entry cost and the reserve price are common knowledge.

2.2. Manipulation of contract terms

Before announcing the auction the procurer sets requirements for the companies and chooses terms of the public contract. Let us assume that the procurer knows several non-price characteristics of one company that distinguish it from the others⁶. Then the procurer decides whether to make entry into the auction possible to everyone or only to company 1. We denote the procurer's strategy “to manipulate” (contract terms) as M and the strategy “not to manipulate” as NM . Further we consider a simple example of the gasoline procurement, which shows how contract terms affect company's entry in auctions.

Figure 1
Manipulation of contract terms



Source: Nizhnii Novgorod, Russia. 2GIS. (http://2gis.ru/n_novgorod)

⁶ Hereinafter we refer to this company as company 1.

One public procurer (an university) wants to buy gasoline at local gas stations. Figure 1 shows nine gasoline stations that belong to the five companies and are situated near the procurer in Nizhnii Novgorod. Each company has a set of certain characteristics that are usually not related to the contract execution, e.g. the number of gasoline stations and their addresses or whether a company is a vertically integrated company or an independent gasoline station. Let us assume that the university has complete information about several unique characteristics of company 1 that distinguish it from competitors. For example, only company 1 has two gasoline stations in this and the neighboring districts of Nizhnii Novgorod, while the other companies have only one station. Then if the procurer adds the condition “the supplier has to have two and more gasoline stations in district X and district Y” (by choosing the strategy M), only company 1 can enter the auction. In contrast, if the procurer does not set such manipulative conditions (NM), each company can enter the auction.

2.3. Corruption subgame

The nature defines the type σ of the public procurer. If $\sigma = 0$ the procurer is benevolent to the society and cannot demand a bribe. If $\sigma = 1$ the procurer is opportunistic and potentially corrupt, so he can demand a bribe⁷.

The procurer if corrupt may propose company 1 a corrupt deal $\langle B, r \rangle$: company 1 gives a bribe B in exchange for the public contract at the reserve price. This deal makes company 1 a favorite company⁸. For the realization of this deal the procurer manipulates contract terms, «tailoring» them to company 1⁹ that prevents the entry of all other companies into the auction. Company 1 learns contract terms of announced auction and understands that she is the only company that can enter. Hence, she makes the highest possible bid and becomes the winner. After the contract is made, company 1 gives a bribe and executes a public contract fearing severe sanctions by the procurer. We consider this reduced-form game as a part of a bigger game where the procurer punishes a company if she violates the terms of a corrupt deal. Corrupt deals are usually a by-product of legal transactions that helps parties to overcome commitment

⁷ In the general case the parameter σ shows which share of the total corruption the public procurer can appropriate as the bribe (Auriol, 2006). To make a corrupt deal the company carries out some organizational costs, for instance, guarantees the secrecy of the deal and transfers the money to the procurer's bank account. Therefore the procurer gets only a certain share of these costs as the bribe. If this share is high, the bribe is big; hence, the procurer is easily bribed. If this share is low (close to zero), the bribe is very low, hence, it is impossible to bribe the procurer.

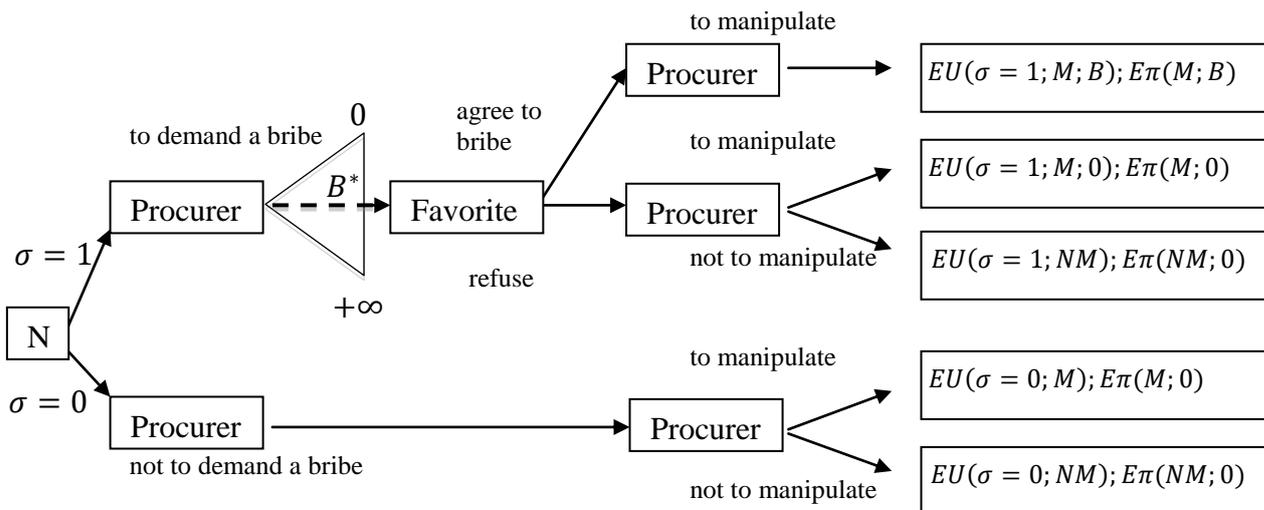
⁸ Problems of corrupt contract performance are beyond this work. One may find more details, e.g., in Lambsdorff, 2002.

⁹ An alternative approach to model manipulation is to assume the corrupt procurer sets such contract terms that no company can meet. For instance, he can request the delivery of large amount of gasoline in a very short period of time that no company can execute because of certain production constraints. Then the procurer tells company 1 that he is ready to “close eyes” on her misfit if she agrees to give a bribe. This way of modeling the manipulation of contract terms is close to the idea that the procurer can distort the quality assessment proposed by Burguet and Che (2004).

(Lambsdorff, 2002), so if a company refuses to give a bribe and execute a public contract, she loses much more than in case when she commits to the corrupt deal. The procurer can use his connections to such public entities, as tax, fire or labor safety department, which initiate many time-consuming tests and impose very high fines for any minor violation, or collude with other public entities to deprive a company from any public contract in the future.

For the sake of simplicity we assume that the procurer has all the bargaining power. He makes take-it-or-leave-it offer to company 1 maximizing the bribe he can extract. In our model the corrupt deal happens before company 1 realizes her production costs. This situation takes place, for instance, when the procurer invites the company that is somehow affiliated with him (e.g. a company run by his former classmates or colleagues) to enter a public auction. This company becomes a newcomer in the public procurement. She realizes the exact value of her production costs after the procurer's invitation, but before the auction (as we follow Samuelson and consider simple and homogeneous product).

Scheme 1
Corruption game and manipulation of contract terms



Notes. B^* is the bribe; M is the strategy «manipulate»; NM is the strategy «not to manipulate».

2.4. Payoffs

Unlike Samuelson's model (1985) and the other preceding papers on auctions with endogenous entry, we consider the public procurer as a separate player with his own utility function EU . This utility may depend on the contract price in two different ways, as far as the procurer may pay the whole contract price $P, P > 0$ or pay nothing if he is financed by the society (or the benevolent government). In what follows we will say that in the former situation the procurer manages his own funds ($\beta = 1$), and in the latter that he manages public funds ($\beta = 0$). As far as we use a common definition of corruption as "the abuse of public office for

the private gain” [Mauro, 1998, p. 11], if the procurer receives a bribe from company 1, corruption arises independently of who is funding the contract. To make the purchase always beneficial for the procurer, we impose the following restriction on the reserve price: $\beta r < 1$.

One may consider this situation in terms of monetary incentives in the public sector, namely, the introduction of a pay-for-performance (Meyer, 1975). Although the output of the public officer is often treated as unobservable and costly to be measured (Dixit, 2002), when this condition does not hold a pay-for-performance may lead to greater improvement in the productivity (Peter K. Lindenauer et al., 2007). In the analyzed situation the contract price may indicate whether the procurer is motivated to cut public expenditures or not. Hence, if $\beta = 1$, the motivation scheme includes the pay-for-performance component; if $\beta = 0$, this scheme is not implemented.

As the result, if the purchase takes place (auction is not void), the expected utility of the procurer EU equals to:

$$EU = Prob(purchase)(1 - \beta EP + \sigma B), \quad (1)$$

where $Prob(purchase)$ is the probability that the purchase takes place, 1 is the contract value for the procurer, EP is the expected contract price.

In equilibrium other companies know about corrupt deal between the procurer and company 1. However the public character of this information does not influence strategies chosen by the procurer, company 1 and other companies, since we do not model the punishment for corruption or monitoring. When the procurer manipulates contract terms, no one, but company 1, can enter the auction and incur entry costs. If company 1 enters, she has the same incentives to make the highest possible bid maximizing her profit. In contrast, when the procurer does not manipulate contract terms, each company makes a bid based on her production costs. The type of the procurer organizing auction (benevolent or corrupt) has no impact on it.

Expected profits of all companies, if no corrupt deal is made, are equal to standard payoffs in English auction with the entry cost:

$$E\pi_i = \begin{cases} 0, & \text{if } i \text{ does not enter} \\ Prob(P_i = \min\{P_1, \dots, P_n\})(P_i - c_i) - k, & \text{if } i \text{ enters} \end{cases} \quad (1.2)$$

where $Prob(P_i = \min\{P_i, \dots, P_n\})(P_i - c_i)$ is the probability that the company i wins the auction.

If the corrupt deal is made, the expected profit of the favorite company equals the following:

$$E\pi_1(B) = r - c_1 - k - B, \quad (1.3)$$

while other companies get zero payoff.

2.5. Timing

All players are risk neutral. The table 1 presents the timing of the game that consists of five active steps: corruption proposal (t=1), consent or refusal to give a bribe (t=2), preparation of contract terms (t=3), entry of companies (t=4) and contract award (t=5). In case of the benevolent procurer the game starts from the Step 3.

Table 1	
Timing of the game	
1	<i>Corruption proposal</i> The procurer if corrupt demands a bribe from company 1 in exchange for the contract at the reserve price (take-it-or-leave-it offer).
2	<i>Consent or refusal to give a bribe</i> If the procurer has demanded a bribe, company 1 decides whether to give it.
3	<i>Preparation of contract terms</i> The procurer sets contract terms: manipulate (M) or not (NM).
4	<i>Entry of companies</i> Each company learns contract terms and decides whether to enter the auction.
5	<i>Contract award</i> The procurer organizes the reverse first-price sealed-bid auction. All players receive zero payoffs if no one enters the auction.
	<i>Outcomes</i> The procurer purchases a product if the purchase has taken place. Company 1 gives a bribe if the corrupt deal has been made.

In the Section 3 we solve the basic model. First, we find the solution of the subgame with the benevolent procurer ($\sigma = 0$) and then examine when favoritism (a set of strategies “demand a bribe; give a bribe”) is the equilibrium in the subgame with the corrupt procurer ($\sigma = 1$).

3. Basic model of endogenous entry in public procurement

3.1. The case of the benevolent procurer

This subgame consists of three active steps: preparation of contract terms (t=3), entry of companies (t=4) and contract award (t=5). In the beginning the benevolent procurer sets contract terms (manipulative or not) that maximize his expected utility and announces the auction. Then each company decides whether she enters the auction depending on the reserve price, the entry cost and her private production costs. If a company enters the auction, she makes a bid maximizing her expected profit. We will find SPNE using the reverse induction.

At the last step (t=5) the procurer organizes a reverse first-price sealed-bid auction and gets the following expected price depending on whether he has manipulated contract terms:

$$P = \begin{cases} Ec_{(1)} + \sum_{i=1}^n E\pi_i + nF(c^*)k, & \text{if the procurer does not manipulate} \\ F(r - k)r, & \text{if the procurer manipulates contract terms} \end{cases} \quad (2)$$

If the procurer has not manipulated contract terms, all companies with production costs below the threshold level c^* enter the auction (Samuelson 1985, 54). If one or more companies enter the auction, the expected contract price is the sum of the minimum expected production costs $Ec_{(1)}$, the expected total profit of n companies $\sum_{i=1}^n E\pi_i$ and the sum of expected entry costs incurred by companies $nF(c^*)k$ (Samuelson 1985, 56). If the procurer has manipulated contract terms in favor of company 1, the contract price is equal to the reserve price r , as company 1 makes the biggest possible bid to maximize her profit. She enters when her production costs are lower than the difference between the reserve price and the entry cost.

At the previous step ($t=4$) each company decides whether she will participate in the auction. This decision, first of all, depends on whether she meets the contract conditions which are the result of the procurer's choice to manipulate contract terms or not.

- The procurer does not manipulate contract terms, NM

Following Samuelson we assume that if the procurer does not manipulate contract terms, each company enters the auction if her production costs do not exceed a certain threshold level c^* (Samuelson 1985).

If the production costs of the company are equal to the threshold, her expected profit in the auction is equal to zero. This company can win the auction only when she is the only bidder, so to make the highest possible bid in the auction (the reserve price) is the dominant strategy for her. Then the entry cost equals the expected auction revenue for this company¹⁰:

$$[1 - F(c^*)]^{n-1}(r - c^*) = k, \quad (3)$$

where $[1 - F(c^*)]^{n-1}$ is the probability that the company with production costs c^* is the only bidder in the auction.

Expected profits of companies participating in the auction depend on their production costs. The lower the production costs are, the greater the probability that a certain company wins the auction is, and the higher her expected profit is. In general, the expected profit of the company with production costs $x < c^*$ equals the profit of the company with production costs equal to the threshold value c^* plus the difference between her production costs x and the second minimum production costs of her competitor in case she wins the auction.

According to the equation (3), the first term is zero, so the company with production costs x gets the expected profit:

¹⁰ For more details see Samuelson (1985).

$$\pi(x) = \int_x^{c^*} [1 - F(x)]^{n-1} dc.$$

The expected profit of the company before realizing the exact value of her production costs equals:

$$E\pi_i = \int_0^1 f(x) \left[\int_x^{c^*} [1 - F(x)]^{n-1} dc \right] dx, i = 1 \dots n.$$

If the production costs are higher than c^* , the profit of the company equals zero. Hence, we replace the upper limit of integration to c^* . Further, integrating by parts and using Leibniz integral rule we get the following:

$$E\pi_i(NM) = \int_0^{c^*} F(c)[1 - F(c)]^{n-1} dc, i = 1 \dots n. \quad (4.1)$$

Total expected profit of companies equals $\sum_{i=1}^n E\pi_i(NM) = nE\pi_i(NM)$.

- The procurer manipulates contract terms, M

In this case all companies, but company 1, cannot enter the auction and receive zero payoffs. If company 1 enters the auction, she makes a bid equal to the reserve price and wins the auction without competition. Expected profits of companies equal:

$$E\pi_1(M) = F(r - k)(r - k) - \int_0^{r-k} f(c)cdc, \quad (4.2)$$

$$E\pi_{j \neq 1}(M) = 0, j = 2, \dots, n,$$

where $\frac{1}{F(r-k)} \cdot \int_0^{r-k} f(c)cdc$ is the conditional expectation of the production costs of company 1 in case she agrees to execute the contract at the reserve price.

At the previous step ($t=3$) the benevolent procurer sets contract terms, which give him maximum utility (see equation (1)):

$$EU = \max \{EU(M; 0); EU(NM; 0)\}.$$

The benevolent procurer compares his expected utilities choosing between the strategy “manipulate” and the strategy “not to manipulate”. If the procurer manipulates contract terms, the purchase takes place with probability $F(r - k)$, when company 1 agrees to execute the contract at the reserve price. Hence, the procurer gets the following expected utility:

$$EU(M; 0) = F(r - k)(1 - \beta r), \quad (5.1)$$

If the procurer does not manipulate contract terms, the purchase takes place with probability $1 - [1 - F(c^*)]^n$, when at least one company out of n companies enters the auction.

As the equation (2) shows, the expected contract price equals the sum of the expectation of the first order statistics and the total profit of companies. As the result, the procurer gets the following expected utility:

$$EU(NM; 0) = 1 - [1 - F(c^*)]^n - \beta \int_0^{c^*} cdG(c) - \beta nE\pi_i(NM) - \beta nF(c^*)k, \quad (5.2)$$

where $\int_0^{c^*} cdG(c)$ is the expected production costs of the winner, $G(c) = 1 - [1 - F(c)]^n$ is its cumulative distribution function.

We calculate the contract price in the equation (5.2) completely in accordance with Samuelson's model. Since our model treats the procurer as a separate player benefiting from the contract, his utility also contains the expected contract value.

The procurer manipulates contract terms if this strategy simply gives him higher expected utility than the strategy “not to manipulate”: $EU(\sigma = 0; M) > EU(\sigma = 0; NM)$. Previously, we assumed that the production costs of companies are uniformly distributed, $c_i \sim U[0; 1]$. Then, substituting the values of the expected utilities of the procurer from equations (5.1) and (5.2) into the inequality (6) and simplifying the expression, we get the condition under which the benevolent procurer manipulates contract terms:

$$(r - k)(1 - \beta r) > (1 - [1 - c^*]^n) \left(1 - \beta \frac{Ec_{(1)} + nE\pi_i(NM) + \beta nF(c^*)k}{1 - [1 - c^*]^n}\right) \quad (6)$$

Let us now examine the choice of the procurer depending on whether he manages public money or private money.

- The procurer manages the public money, $\beta = 0$

In this case the expected utility of the procurer equals the probability that the purchase takes place. Therefore, if the probability that company 1 agrees to execute the contract at the reserve price is higher than the probability that the auction is not void, the procurer manipulates contract terms. If this condition does not hold, the opposite is true.

Proposition 1

The benevolent procurer managing the public money does not manipulate contract terms if the reserve price is lower than 1, and manipulates contract terms if the reserve price exceeds 1.

Proof

By substituting the value of the entry cost from the equation (3) into the expression (6) and simplifying it, we find under what conditions the benevolent procurer chooses the strategy «manipulate»:

$$EU(M; 0) > EU(NM; 0),$$

$$r - k > 1 - [1 - c^*]^n,$$

$$r - [1 - c^*]^{n-1}(r - c^*) > 1 - [1 - c^*]^n,$$

$$r(1 - [1 - c^*]^{n-1}) > 1 - [1 - c^*]^{n-1},$$

Since $k > 0$, $[1 - c^*]^{n-1} < 1$, we divide both sides of the inequality by $1 - [1 - c^*]^{n-1}$

and get:

$$r > 1 \tag{7}$$

Proposition 1 is proved.

We find that the decision of the benevolent procurer upon manipulating contract terms depends only on the level of the reserve price. This decision is not related to the entry cost and the number of companies in the market. Figures 2a – 2b in Appendix 1 illustrate this result. One can observe that for different number of companies in the market ($n = 2, n = 5$) and different entry cost ($k = 0.1, k = 0.4$) the found relationship between expected utilities of the benevolent procurer when he chooses “to manipulate” and “not to manipulate” holds. If the reserve price is lower than 1, the benevolent procurer does not manipulate contract terms; if the reserve price is higher than 1, he manipulates them.

- The procurer manages the private money, $\beta = 1$

Proposition 2

The benevolent procurer managing the private money does not manipulate contract terms if the reserve price is lower than 1 or if the price increase has higher effect on the procurer's utility than the purchase probability.

Proof

At first we examine the case when the reserve price is lower than the maximum production costs: $r < 1$. In this case the first multiplier on the left side of the inequality (6) is always lower than the first multiplier on the right side of this inequality:

$$(r - k) < (1 - [1 - c^*]^n).$$

According to the rules of the auction with a reserve price, the winning bid (the contract price) cannot exceed the reserve price:

$$r > Ec_{(1)} + nE\pi_i(NM) + \beta nF(c^*)k.$$

Hence, the second multiplier on the left side of the inequality (6) is always lower than the second multiplier on the right side of this inequality. Thus if the procurer manages the private

money he does not manipulate contract terms if the reserve price is lower than 1. We have proved the first part of Proposition 2.

Then we examine the case when the reserve price is higher the maximum production costs: $r > 1$. In this case the first multiplier on the left side of the inequality (6) is always higher than the first multiplier on the right side of this inequality. However the second multiplier on the left side of the inequality (6) is always lower than the second multiplier on the right side of this inequality. If the procurer switches from the strategy “not to manipulate” on the strategy “to manipulate”, he raises the probability of the purchase that results in higher expected utility, but also increases the expected contract price that lowers his expected utility.

One may re-write the inequality (6) in the following way:

$$EU(\sigma = 0; M) > EU(\sigma = 0; NM),$$

$$\frac{r-k}{1-[1-c^*]^n} > \frac{(1-\beta r)}{(1-\beta EP)} \quad (8)$$

Thus, the choice of the procurer depends on the relationship of two opposite effects:

1. the effect of the purchase probability: $(r - k)/(1 - [1 - c^*]^n)$,
2. the effect of the price increase: $(1 - \beta r)/(1 - \beta EP)$.

If the effect of the purchase probability exceeds the effect of the price increase, the procurer gets higher expected utility manipulating contract terms: $EU(\sigma = 0; M) > EU(\sigma = 0; NM)$. Otherwise the procurer does not manipulate contract terms.

Proposition 2 is proved.

3.2. The case of the corrupt procurer

The subgame with the corrupt procurer consists of five active steps: corruption proposal (t=1), consent or refusal to give a bribe (t=2), preparation of contract terms (t=3), entry of companies (t=4) and contract award (t=5). Using the reverse induction we check when the set of strategies “demand a bribe, give a bribe” is SPNE. In other words, we find the conditions leading to favoritism in equilibrium. Steps 4 and 5 in this subgame corresponds to the respective steps in the subgame with the benevolent procurer, since bids and entry of the companies depend not on the type of the procurer (benevolent or corrupt), but on the manipulation of contract terms. Therefore we skip these steps and focus on steps 1, 2 and 3 (see Table 1).

At the step 3 the procurer sets contract terms. If the company refuses to give a bribe, agents get the same payoffs as in the subgame with the benevolent procurer. From Propositions 1 and 2, the reserve price and the financial source (public or private money) affect the procurer's choice to manipulate or not. If company 1 agrees to give a bribe, the corrupt procurer

manipulates contract terms. Then the expected utility of the procurer and the expected profit of company 1 equal, respectively:

$$EU(M; B) = 1 - \beta r + B, \quad (9.1)$$

$$E\pi_1(M; B) = r - \int_0^1 f(c)cdc - k - B. \quad (9.2)$$

Expected profits of other companies equal zero.

At the step 2 company 1 decides to agree on the corrupt deal or not. If it is more beneficial to the procurer to manipulate contract terms even without bribe ($EU(M; 0) \geq EU(NM; 0)$), company 1 has no incentives to bribe him. This case is simple and less interesting than the case when the corrupt deal is possible. In order to concentrate on the latter case it is enough to make an assumption that the reserve price is lower than the maximum production costs: $r < 1$ (Propositions 1 and 2). Then if company 1 refuses to give a bribe, the procurer does not manipulate contract terms. This may encourage company 1 to agree on a corrupt deal, and favoritism will arise.

Company 1 gives a bribe if she gets at least the same expected profit as in the case when she refuses (incentive compatibility constraint, IC_1) and its expected profit is not lower than zero (individual rationality constraint, IR_1):

$$\begin{cases} E\pi_1(M; B) \geq E\pi_1(NM; 0), & (IC_1) \\ E\pi_1(M; B) \geq 0 & (IR_1) \end{cases} \quad (10)$$

If the incentive compatibility constraint holds, the individual rationality constraint is always true, since we get from the equation (4.1) that $E\pi_1(NM; 0) > 0$. Therefore in what follows we may ignore this condition. Since the procurer has all the bargaining power and extracts the highest possible bribe that company 1 can give, incentive compatibility constraint holds as the equality. Let us examine it in more detail.

If company 1 refuses to bribe, the procurer does not manipulate contract terms (this results from Propositions 1, 2 and an assumption that $r < 1$). Comparing the expected profits of company 1 in situations when the procurer demands a bribe, manipulates contract terms and does not manipulate them (equations 4.1 and 9.2), we get the following:

$$\begin{aligned} E\pi_1(M; B^*) &= E\pi_1(NM; 0) \Rightarrow \\ r - \int_0^1 f(c)cdc - k - B &= \int_0^{c^*} F(c)[1 - F(c)]^{n-1}dc \end{aligned}$$

The bribe is by definition higher than zero, therefore taking into account that $c_i \sim U[0; 1]$ we simplify the expression given above and find another constraint for the reserve price:

$$\begin{aligned}
B^* &= r - \int_0^{c^*} c[1-c]^{n-1}dc - k - \frac{1}{2}, \\
B^* &> 0, \\
r &> \int_0^{c^*} c[1-c]^{n-1}dc + k + \frac{1}{2}
\end{aligned} \tag{11}$$

Inequality (11) shows that favoritism arises only if the reserve price is higher than the sum of the expected profit of a company in the auction without manipulation, the entry cost and the expected production costs of a company in the beginning of the game (in case of the uniform distribution, $c_i \sim U[0; 1]$, they equal $\frac{1}{2}$).

Substituting the expression for the entry cost from the equation (3) we get the highest bribe that company 1 agrees to give:

$$B^* = \max B = r(1 - [1 - c^*]^{n-1}) + c^*[1 - c^*]^{n-1} - \int_0^{c^*} c[1-c]^{n-1}dc - \frac{1}{2} \tag{12}$$

Previously we have found out that if the reserve price is low (see inequality (11)) or very high (see Propositions 1, 2), favoritism does not arise. In the former case the bribe is negative, while in the latter case company 1 has no incentives to bribe the procurer, because he manipulates contract terms even without a bribe. Meanwhile favoritism may occur when reserve prices lie between two these levels. Hereinafter we call *intermediate reserve price* each reserve price that lies in the interval $(\int_0^{c^*} c[1-c]^{n-1}dc + k + \frac{1}{2}; 1)$. We wonder what factors affect the size of the optimal bribe B^* when reserve prices are intermediate.

We hypothesize that the bribe positively depends on the number of companies in the market and the reserve price and negatively depends on the entry cost. All these factors increase the gap between the expected profit of company 1 from the auction with and without manipulation, thus stimulating her to give a higher bribe for keeping a monopoly position.

First, the lower the entry cost is, the higher is the difference between the expected profit of company 1 from the auction with and without manipulation. On the one hand, company 1 always enters alone in the auction with manipulation, so decrease in the entry cost raises the probability of her entry, but does not affect the price she gets. So lowering entry cost is very beneficial for company 1. On the other hand, company 1 faces competition in the auction without manipulation. Despite lower entry cost raises the probability of her entry, it always raises the probability of entry of more competitors, so the probability that company 1 wins the auction may decrease. Also in case when $c_i \sim U[0; 1]$ the expected price increases in the entry cost, so if the entry cost decreases, company 1 makes lower bid that reduces her expected profit in the auction without manipulation. In order to avoid this situation company 1 agrees to give higher bribe.

Second, the more companies are in the market, the lower expected profit one company gets from entering the auction without manipulation. In order to avoid competition and maintain

a monopoly position, company 1 agrees to give higher bribe. One can easily notice it by comparing the expected profit in case of monopoly ($T = M$) with expected profits in cases of two other market structures: oligopoly ($T = NM; N \ll +\infty$) and perfect competition ($T = NM; N \rightarrow +\infty$). If there is monopoly, company 1 is the only entrant, so she always wins the auction at the highest reserve price. If there is perfect competition, the number of companies in the market tends to infinity and the expected profit of each company in the beginning of the game tends to zero. If there is oligopoly, the number of companies in the market is lower. Fewer companies enter the auction in comparison with the perfect competition case, hence, each company wins the auction with higher probability and at higher price (e.g. see Krishna, 2002, 21). The shift from monopoly to perfect competition is associated with more losses for company 1, than the shift from monopoly to oligopoly. Hence, in the former case company 1 gives higher bribe to maintain the monopoly position, than in the latter. Generally, the more companies are in the market, the more severe losses company 1 faces in the auction without manipulation and the higher is the bribe to prevent these losses.

Third, higher the reserve price is, the higher is the difference between the expected profit of company 1 from the auction with and without manipulation. The intuition of this relationship is analogous to the relationship between the reserve price and the bribe we have considered earlier. The general idea is that the reserve price always makes entry in the auction with manipulation more beneficial for company 1; it may have positive impact for the expected profit in the auction without manipulation, but this impact is lower because company 1 wins this auction with much lower probability and at lower price.

Proposition 3

The optimal bribe increases in the reserve price and the number of companies in the market and decreases in the entry cost.

Proof of the Proposition 3 is in Appendix 2. The Table 2 demonstrates main results.

Table 2			
Determinants of the optimal bribe			
<i>Factor</i>	The entry cost, k	The number of companies, n	The reserve price, r
<i>Sign of the derivative</i>	Negative	Positive	Positive

Proposition 4

Let assume that the market consists of two companies, $N = 2$. Then there is a unique size of the entry cost \tilde{k} when the optimal bribe equals zero for each intermediate reserve price:

$$B^*(\tilde{k}) = \frac{1}{3}(c^*(\tilde{k}))^3 - \frac{3}{2}(c^*(\tilde{k}))^2 + (r+1)c^*(\tilde{k}) - \frac{1}{2} = 0, \quad (14)$$

$$\text{for } \forall r, r \in \left(\int_0^{c^*} c[1-c]^{n-1}dc + k + \frac{1}{2}; 1\right).$$

If the entry cost is below this level, company 1 gives a positive bribe. If the entry cost exceeds this level, company 1 is not ready to give a positive bribe and the corrupt deal never takes place.

Proof of the Proposition 4 is in the Appendix 3.

One may interpret the Proposition 4 in the following way. When the reserve price is intermediate, lowering the entry cost does not lead to favoritism if the magnitude of their decrease is small. On the contrary, lowering the entry cost stimulates favoritism if the magnitude of this decrease is large enough. So the governmental attempts to substantially decrease the entry cost may provoke favoritism and prevent entry of honest companies.

Now we turn back to the general case with $n \geq 2$ companies in the market.

At the Step 1 the procurer decides whether to demand a bribe B^* . He demands a bribe if it leads to higher expected utility (incentive compatibility constraint, IC_P), and his expected utility is not lower than zero (individual rationality constraint, IR_P):

$$\begin{cases} EU(M; B) > \max \{EU(NM; 0); EU(M; 0)\} & (IC_P) \\ EU(M; B) \geq 0 & (IR_P) \end{cases} \quad (15)$$

If the incentive compatibility constraint holds, the individual rationality constraint is always true, since equations (5.1-5.2) result in $EU(M; 0), EU(NM; 0) > 0$. Therefore in what follows we may ignore this condition. Let us examine the incentive compatibility constraint in more details. Since we consider the case when favoritism is possible, $r < 1$, hence, the benevolent procurer does not manipulate contract terms (from this assumption mentioned above and the inequality (6)). Then we may re-write IC_P in the following way: $EU(M; B) > EU(NM; 0)$.

Proposition 5

The expected utilities $EU(M; B)$, $EU(NM; 0)$ decrease in the entry cost. The higher the entry cost is, the higher contract price the procurer pays in the auction without manipulation and the lower the optimal bribe is in case of manipulation.

Proof of the Proposition 5 is in the Appendix 4.

Lower entry cost decreases the expected utility of the corrupt procurer independently of what set of strategies (“demand a bribe, manipulate” or “not to demand a bribe, not to manipulate”) he chooses. We wonder how the entry cost affects the relationship between these two expected utilities. The first possibility is that the expected utility of one set of strategies is always higher than the expected utility of the other set. Then the procurer always makes the former choice. The second possibility is that under different levels of the entry cost the gap between the expected utilities of the procurer is different. Then if the entry cost changes around some threshold level, the procurer switches from one set of strategies to the other one.

We define the difference between two expected utilities of the procurer as ΔU : $\Delta U = EU(M; B) - EU(NM; 0)$. From expressions (5.2) and (9.2) we get that this difference equals:

$$\Delta U = (1 - \beta)r + (1 - r + \beta nc^*(r - c^*)) [1 - c^*]^{n-1} + (2\beta n - 1) \int_0^{c^*} c [1 - c]^{n-1} dc - \frac{1}{2} \quad (16)$$

Under $\Delta U > 0$ the procurer chooses a set of strategies “to demand a bribe, to manipulate” if the favorite is ready to give a possible bribe. Under $\Delta U < 0$ the procurer chooses a set of strategies “not to demand a bribe, not to manipulate” independently of company’s 1 readiness to give a bribe. In the latter case even the highest possible bribe does not compensate the procurer’s losses when he manipulates contract terms instead of organizing the auction without manipulation.

Further we research the choice of the procurer depending on whether he manages public money or private money. As previously, we focus on the simple case with two companies in the market, $n = 2$.

- The procurer manages the public money, $\beta = 0$

Proposition 6

If the procurer manages the public money, the difference between procurer’s utilities is positive, $\Delta U > 0$. The procurer always benefits from favoritism, and favoritism always arises when company gives a positive bribe.

Proof

Substituting $n = 2$ and $\beta = 0$ into the equation (16) we get:

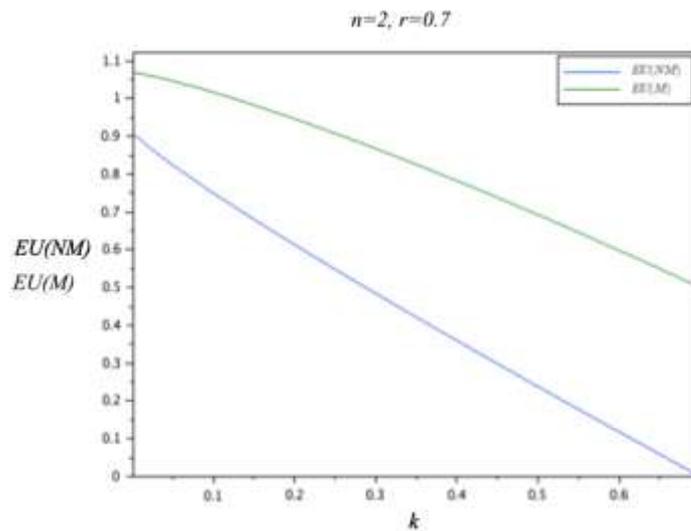
$$\Delta U = \frac{1}{3}(c^*)^3 - \frac{1}{2}(c^*)^2 - (1-r)c^* + \frac{1}{2} \quad (17)$$

We equate the difference between procurer's utilities to zero. Under previously imposed assumptions, $c^* < r$, $c^* < 1$, $r \in (\int_0^{c^*} c[1-c]^{n-1}dc + k + \frac{1}{2}; 1)$, the equation does not have real roots. The procurer's expected utility of a set of strategies "to demand a bribe, to manipulate" exceeds the procurer's expected utility of a set of strategies "not to demand a bribe, not to manipulate" for any reasonable level of the entry cost. Hence, when company 1 is ready to give a positive bribe, the corrupt procurer always demands it if he manages the public money.

Proposition 6 is proved. The Figure 5 illustrates expected utilities of the procurer when the reserve price equals 0.7.

Figure 5

Expected utilities when procurer manipulates and does not manipulate contract terms



- The procurer manages the private money, $\beta = 1$

Proposition 7

If the procurer manages the private money, the difference between procurer's utilities is zero or negative, $\Delta U \leq 0$. The procurer never benefits from favoritism, and favoritism does not arise.

Proof of the Proposition 7 is in the Appendix 5.

4. Impact of the entry cost on contract prices

In this section we analyze how the entry cost influences the contract price in more details. As we have shown before (see Propositions 1-3, 5-6 and equation (15)), the contract prices paid by the corrupt procurer ($\sigma = 0$) and the benevolent one ($\sigma = 1$) depend on the reserve price, the entry cost and the number of companies in the market:

I. $r > 1$ (high) \Rightarrow

$$EP(\sigma = 0) = EP(\sigma = 1) = r,$$

II. $1 > r > \int_0^{c^*} c[1-c]^{n-1}dc + k + \frac{1}{2}$ (intermediate) \Rightarrow

$$EP(\sigma = 0) = 2n \frac{\int_0^{c^*} c[1-c]^{n-1}dc}{1-[1-c^*]^n} + \frac{nc^*k}{1-[1-c^*]^n}$$

$$EP(\sigma = 1) = \begin{cases} r, & \text{in case of favoritism} \\ EP(\sigma = 0), & \text{otherwise} \end{cases}.$$

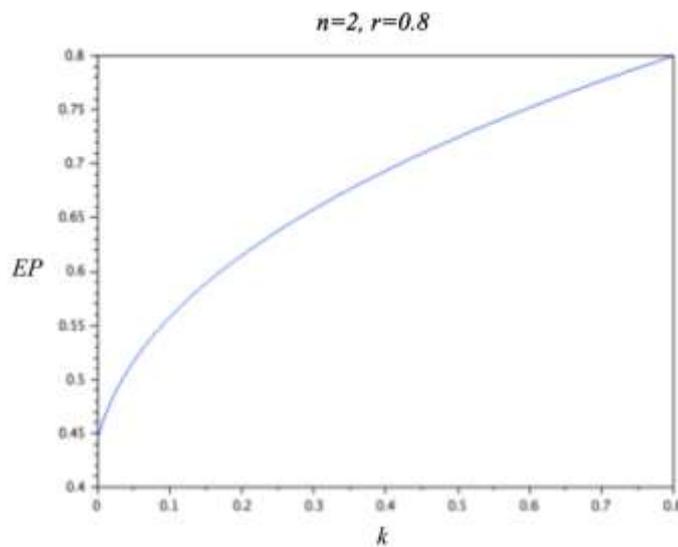
III. $\int_0^{c^*} c[1-c]^{n-1}dc + k + \frac{1}{2} > r$ (low) \Rightarrow

$$EP(\sigma = 0) = EP(\sigma = 1) = 2n \frac{\int_0^{c^*} c[1-c]^{n-1}dc}{1-[1-c^*]^n} + \frac{nc^*k}{1-[1-c^*]^n}$$

Cases I and III demonstrate that if reserve prices are high or low, contract prices paid by the corrupt and the benevolent procurers are the same, while case II shows that if the reserve price is intermediate, contract prices may be different. We are mostly interested in the case II, because it is more appropriate to the procurement practice when favoritism and manipulation is a possible equilibrium, but not the only one. So further we analyze the influence of the entry cost on the contract price paid by each type of the procurer when the reserve price is intermediate. Again we examine the case with two companies in the market.

4.1. The case of the benevolent procurer

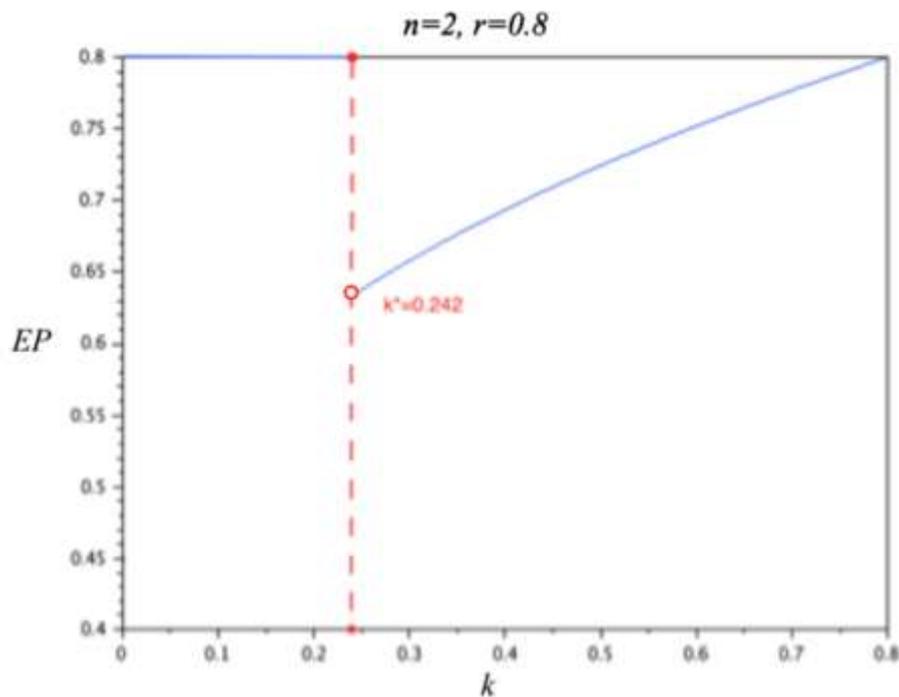
Under accepted assumptions the benevolent procurer does not manipulate contract terms independently on the financial source (public money or private money). The derivative of the expression (8.2) with respect to the entry cost when $n=2$ is strictly higher than zero. Hence, exogenous decrease in the entry cost leads to lower expected contract price paid by the benevolent procurer. Figure 6 shows this relationship if the reserve price equals 0.80.

Figure 6**Expected contract price paid by the benevolent procurer, $\beta = 0$** 

4.2. The case of the corrupt procurer

Unlike the case of the benevolent procurer, the financial source influences the corrupt procurer's decision to manipulate contract terms. First, if the procurer manages the public money, favoritism may arise and the procurer manipulates contract terms. When the entry cost is high enough (exceed the certain threshold level, see Proposition 6), company 1 cannot give a bribe. Hence, the corrupt procurer does not manipulate contract terms and gets the same contract price, as the benevolent one. When the entry cost is decreasing, the optimal bribe increases (Proposition 3), and when the entry cost is lower than the certain threshold level company 1 becomes ready to bribe the procurer. Since the procurer manages the public money, he always benefits from favoritism (Proposition 6). Then the procurer makes a corrupt deal with the company 1 and receives a bribe in exchange for the contract at the reserve price. Therefore the contract price dramatically increases. This situation corresponds, for instance, to the decrease in the entry cost from 0.3 to 0.2 on the Figure 7.

Figure 7
Expected contract price paid by the corrupt procurer, $\beta = 1$



If the procurer manages the private money, we get the different result. According to the Proposition 7, in this case the strategy “not to manipulate” is always a weakly dominant strategy for the procurer, and no favoritism arises. Hence, the contract price paid by the corrupt procurer equals the contract price paid by the benevolent procurer and always decreases in the entry cost (see Figure 6).

5. Extensions of the basic model

Timing

Our model addresses the situation when company 1 learns her production costs after the corrupt deal, but before the auction. Thus, the considered situation occurs under two following conditions. First, a product is simple, which means that a company learns her production costs before the auction and the entry into the auction is selective. Second, for the sake of simplicity, the procurer and company 1 make a corrupt deal before realization of production costs (there is substantially more time between the corrupt deal and the auction, than between the auction and the start of the contract performance).

Let us assume the opposite: company 1 and the procurer know the production costs of company 1 before the corrupt deal. If these production costs are high, the favorite company cannot give a bribe (or even has no incentives to enter the auction) and the corrupt deal does not

take place. If company 1 incurs such production costs that the optimal bribe exceeds the negative difference between the two expected utilities of the procurer, favoritism arises. Higher entry cost makes entry in the auction with manipulation much less beneficial, thus decreasing the bribe that company 1 is ready to get for winning this auction at the reserve price. Hence, the optimal bribe decreases in the entry cost. If the entry cost exceeds some threshold level (that depends negatively on the realized production costs of company 1), the optimal bribe becomes lower than the negative difference between two expected utilities of the procurer, and favoritism is gone. Thereby the main result of the model is robust to the timing of the corrupt deal.

Bargaining powers of procurer and company 1

Following the convention adopted in the literature (e.g., see. Laffont, 2000), we have assumed that the procurer has all the bargaining power that allows him to demands the highest possible bribe. Now we relax this assumption and examine how it affects the result.

Let us consider the simplest case when the procurer and company 1 have equal bargaining powers and the procurer disagree to manipulate contract terms without a bribe ($r < 1$). Then the new optimal bribe B' is the Nash bargaining equilibrium:

$$F = EU(M; B') \cdot E\pi_1(M; B') \rightarrow \max_{B'} \quad ,$$

s.e.

$$\begin{cases} EU(M; B') > EU(NM; 0) \\ E\pi_1(M; B') > E\pi_1(NM; 0), \\ B' > 0 \end{cases}$$

$$\text{where } EU(M; B') = 1 - \beta r + B';$$

$$E\pi_1(M; B') = r - k - B' - \frac{1}{2};$$

$$EU(NM; 0) = 1 - [1 - F(c^*)]^n - \beta \int_0^{c^*} c dG(c) - \beta n E\pi_i(NM) - \beta n F(c^*) k;$$

$$E\pi_i(NM; 0) = \int_0^{c^*} F(c) [1 - F(c)]^{n-1} dc, i = 1 \dots n \text{ (these expressions result from}$$

equalities (9.1), (9.2), (5.2) and (4.1), respectively).

A new optimal bribe reduces the difference between the expected utilities of the procurer and increases the difference between the expected profits of company 1 from different strategies. Therefore company 1 agrees to give a bribe more often, while the procurer demands it less often compared to the basic model.

F.O.C.

$$\frac{dF}{dB'} = 0, B' = -\frac{(\beta+1)}{2} r + \frac{1}{2} k + \frac{3}{4}.$$

F.O.C. shows that unlike the basic model, the new optimal bribe increases in the entry cost. The relationship between the entry cost and the sustainability of favoritism depends on whether the procurer manages the public money or the private money.

If the procurer manages the public money, $\beta = 0$, then

$$EU(M; B') > EU(NM; 0) \text{ and } B' = -\frac{1}{2}r + \frac{1}{2}k + \frac{3}{4} > 0,$$

hence, favoritism is the equilibrium (as in the basic model).

If the procurer manages the private money, $\beta = 1$, a new optimal bribe is positive if $k > 2r - \frac{3}{2}$. Since $1 > k > 0$, favoritism may become more or less stable in comparison to the basic model depending on other factors.

Endogenous reserve price

We relax the assumption that the reserve price is given exogenously and research how the corrupt and the benevolent procurers choose reserve prices. Let us consider the case when the procurer manages the public money. The maximum expected utility of the benevolent procurer equals:

$$\max EU(\beta = 0; r) = \max \{EU(NM; 0; r); EU(M; 0; r)\},$$

According to equations (5.1) and (5.2), the maximum of this function is reached when the procurer chooses the strategy “to manipulate” and sets the reserve price equals to the sum of the maximum production costs and the entry cost:

$$\max EU(0; r) = EU(M; r = 1 + k) = 1.$$

The maximum expected utility of the corrupt procurer equals:

$$\max EU(\beta = 1; r) = \max \{EU(NM; 0; r); EU(M; 0; r); EU(M; B'; r)\}.$$

According to the equality (9.1) if the procurer manages the public money, his expected utility in case of favoritism positively depends on the optimal bribe: $\frac{dEU(M; B')}{dB'}$. Hence, he has incentives to set the highest possible reserve price that exceeds the optimal reserve price set by the benevolent procurer.

In the following research we examine the difference between reserve prices set by the benevolent procurer and the corrupt procurer when they manage private money.

6. Case-study of Russian gasoline procurement

Public procurement constitutes a substantial share of GDP in Russia and relates to a variety of rent-seeking problems from bid rigging to poor contract performance. Some anecdotal

evidence and speeches of senior officials demonstrate that high public waste is one of the biggest issues in Russian public procurement¹¹. The recent e-procurement reform conducted in 2010-2011 was aimed at reducing prices in public procurement:

“[In the first year after the reform] we expect to save twice as much public spending in e-auctions, approximately 400-500 mln. rubles” – stated one of initiators of the reform, the head of the Federal antimonopoly agency Igor Artemiev¹².

In the case study we examine how the introduction of e-auction influenced entry of companies and prices in public auctions. In terms of our model, this shift from traditional auctions to e-auctions¹³ means the decrease in the entry cost. We consider two indicators of the entry (the total number of companies and the number of companies that submitted bids in an auction) and two indicators of the price decrease (the price discount made by the winner and the relative contract price¹⁴). It is impossible to indicate on corruption without insider information about side payments or revealed corruption cases. However we can illustrate the theoretical model by analyzing different actions of procurers and their outcomes, and make propositions about procurers' incentives on the basis of it. According to the model,

- if the procurer did not manipulate contract terms in e-auctions, more companies entered and prices decreased in e-auctions compared to open-bid auctions;
- if the procurer manipulated contract terms in e-auctions, entry did not change and prices remained the same or increased in e-auctions compared to open-bid auctions.

As e-procurement reform might somehow affect the behavior of procurers (manipulation of contract terms and setting reserve prices) and bidders (tacit collusion), further we take these possible changes into account.

Background

We analyze Russian procurement during the period 2008-2013 when the Federal Law 94-FL “On public procurement” strictly regulated the procurement process. The majority of simple homogeneous goods were purchased through two procurement procedures: open-bid auctions and sealed-bid auctions. Both of them were typical first-price auctions started from the maximum reserve price set by the procurer.

¹¹ E.g. see this publication in the media that states public waste equals one eighth of the consolidated Russian budget: http://www.vedomosti.ru/newspaper/article/248724/ukrast_trillion

¹² <http://pravo.ru/news/view/25397/>

¹³ In what follows we refer to the former type of auction as an open-bid auction and to the latter type of auction as an e-auction.

¹⁴ Following Balsevich and Podkolzina (2014) we calculate the relative contract price as the relation between the final bid made by the winner and the total price of the same contract in average prices measured by the Russian statistical agency (www.gks.ru).

Sealed-bid auctions were always treated as an extra procedure. The procurer might use sealed-bid auction only for small contracts (the reserve price below a certain threshold) once in a quarter. A sealed-bid auction gave procurers wider opportunities for corruption, because bidders submitted their bids in closed envelopes to the procurer or the public commission. Therefore dishonest procurer could open envelopes illegally and used the right of first refusal in exchange for a bribe¹⁵.

Open-bid auctions were treated as a priority procurement procedure in 2008-2013. In 2010 Russian government started e-procurement reform aimed at reducing contract prices by converting traditional open-bid auctions in electronic format. Lower entry cost of companies and anonymity of bidders was expected to be one of key instruments of this reform. First, e-auctions substantially reduced paperwork. Unlike open-bid auctions, which were organized in traditional way in the public procurer's office, e-auctions were organized online on e-platforms chosen by the government. Second, e-auctions concealed identities of bidders, so that each bidder observed bids of his/ her competitors, but did not know who they are. Such anonymous bidding raised monitoring costs of cartels, thus making them more vulnerable.

After 2011 the procurement law obliged public procurers to set reserve prices on the basis of informational sources, e.g. official statistics, letters of the Ministry of Economic Development, market analysis or offers of companies. The list of these informational sources was open and the procurer was free to choose any additional informational source. We address this change in our case analysis and use our theoretical model to demonstrate that the potentially corrupt procurer and the benevolent procurer used different informational sources to set reserve prices that resulted in different contract prices.

Public procurement of gasoline

We choose public procurement of gasoline through gasoline stations organized by two big procurers in Nizhnii Novgorod for the following reasons. First, gasoline¹⁶ is a simple homogenous product that perfectly satisfies main assumptions of our model. The gasoline delivered via gasoline stations has the same level of quality in the public procurement and the private market. Moreover, differences in contract prices organized by different procurers reflect public waste (caused by corruption or collusion) rather than the quality difference.

Second, we need to collect data on open-bid auctions before e-procurement reform (published on regional web-sites) and e-auctions after e-procurement reform (published on the federal web-site, <http://zakupki.gov.ru>). Therefore we choose Nizhnii Novgorod as one of

¹⁵ E.g. see Burguet and Perry (2007); Compte, Lambert-Mogiliansky, and Verdier (2005)

¹⁶ Balsevich and Podkolzina (2014), Yakovlev et al. (2015) use these advantages of gasoline to study such topics, as corruption and repeated interactions between public procurers and suppliers.

regions with a sufficient number of open-bid auctions and e-auctions and transparent regional web-site (<http://www.goszakaz.nnov.ru>).

A typical public contract contains the following parts:

- the subject of the contract: types of the gasoline and their volume (in liters);
- the duration of the delivery: the relevant period when gasoline stations should provide gasoline for the procurer's cars (in days);
- the geographical area: the area where gasoline stations should be located, e.g. certain local areas, districts, cities, regions;
- extra requirements to bidders, e.g. round-the-clock delivery of the gasoline.

Low number of companies and wide spread horizontal collusion are two specific features of the gasoline market which we should take into account in our analysis.

Gasoline auctions: procurer X and procurer Y

We focus on public procurement auctions organized by two public procurers. Procurer X was a big public hospital, while procurer Y was the regional agency of the Ministry of Emergency Situations. Both procurers were situated in the same district of Nizhniy Novgorod close to each other (<3 km between them, see Figure 8) and asked for the gasoline through gasoline stations located in Nizhnii Novgorod and Nizhegorodskaya oblast (hereinafter – the region).

Figure 8
Procurers X and Y and local gasoline stations



Case I: procurer X

Procurer X organized 8 open-bid auctions in 2008-2010 and 13 e-auctions in 2011-2013. One can observe the following changes in contract characteristics between open-bid auctions and

e-auctions. First, the procurer organized e-auctions more often than open-bid auctions; public contracts became smaller at average. The reserve contract price decreased a bit if we take inflation into account (at least 6% a year); there were fewer types and volumes of gasoline in e-auctions contracts at average (see Table 3). Second, the procurer set lower reserve prices in e-auctions than in open-bid auctions. Since 2011 he used the most objective sources of information: regional statistics and official recommendations made by the Ministry of Economic Development. These lower prices might decrease the entry of companies (see Samuelson, 1985).

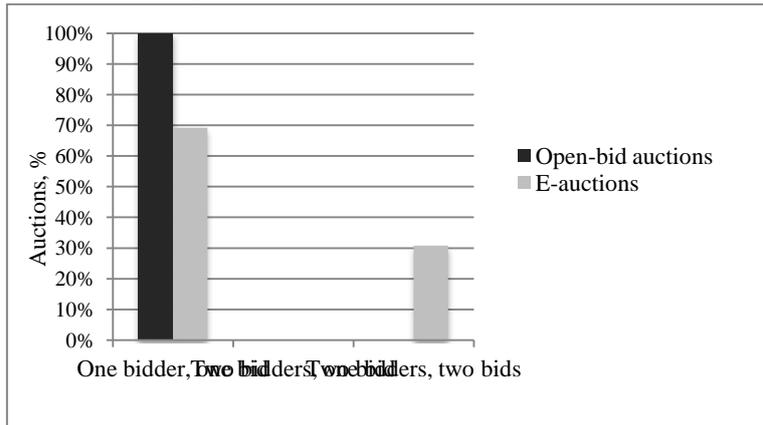
Table 3
Some contract characteristics: Procurer X

Variables	Open-bid auctions		E-auctions	
	Mean	Std. Dev.	Mean	Std. Dev.
Reserve price, rub.	819583.10	509630.3	954937.3	569926.5
Total volume of gasoline	37650	21057.27	33730.77	19482.49
Types of gasoline	3.13	1.36	1.85	.80
24 hours delivery	0	0	0	0
Reserve price/ market price	1.065	.022	1.018	.006
Obs.	8		13	

In our opinion, the procurer did not set strict contract terms in order to prevent some companies from bidding. The duration of the delivery remained reasonable (one, three or six months). Procurer X never asked for round-the-clock delivery of the gasoline. He demanded that gasoline stations should be situated in Nizhnii Novgorod and the region, which seems to be reasonable as ambulance cars were sent to different parts of the region. The requested geographical area remained the same in open-bid auctions and e-auctions.

Hence, two potential factors might affect the entry in different directions: lower reserve price and lower entry cost. The data shows that the entry increased in e-auctions compared to open-bid auctions, so the decrease in the entry cost had stronger effect on the entry. An extra bidder started to enter e-auctions and always made bids (see Table 4). We checked for possible affiliation between two bidders in each auction and did not find anything.

Table 4
Bidding in auctions: Procurer X



Bidders competed more aggressively and prices dropped down (see Table 5). We observe both higher discounts made by the winner and lower relative contract prices. The average contract price was 5.8% higher than the market price in open-bid auctions and was equal to the market price in e-auctions.

Table 5
Entry and prices: Procurer X

Variables	Open-bid auctions		E-auctions	
	Mean	Std. Dev.	Mean	Std. Dev.
Total bidders	1	0	1.31	.48
Bidders that made bids	1	0	1.31	.48
Passive bidding	0	0	0	0
Discount	.006	.006	.018	.008
Relative price	1.058	.024	1.000	.008
Obs.	8		13	

Case II: procurer Y

Procurer Y organized 13 open-bid auctions in 2008-2010 and 12 e-auctions in 2011-2013. Public contracts in open-bid auctions and e-auctions differed in the following way. First, reserve prices and the number of gasoline types in the contract decreased, while the total volume of gasoline in the contract increased in e-auctions compared to open-bid auctions. Such a high difference (see Table 6) results mostly from one huge atypical public contract concluded in 2008 for the whole next year. It was the only public contract with more than one gasoline type in 2008-2010; its reserve price and total volume was approximately 2.5 as big as the average reserve price and total volume of the rest of public contracts made in this period. If we drop this public contract (Table 6, second part), the only crucial change in contract characteristics in the remained public auctions is the decrease in types of gasoline in e-auctions compared to open-bid auctions.

Second, procurer Y set higher reserve prices in e-auctions than in open-bid auctions. Since 2011 procurer Y frequently asked bidders about the desired reserve price, which is the most corrupt and biased source of information. He presented results of two requests as public information and results of three other requests as secret. In the former case the company that procurer Y chose as an informational source later won two e-auctions at higher prices in the absence of active competitors. In the latter case it is impossible to identify the companies that he chose. In the rest of e-auctions procurer Y used regional statistics for setting the reserve price.

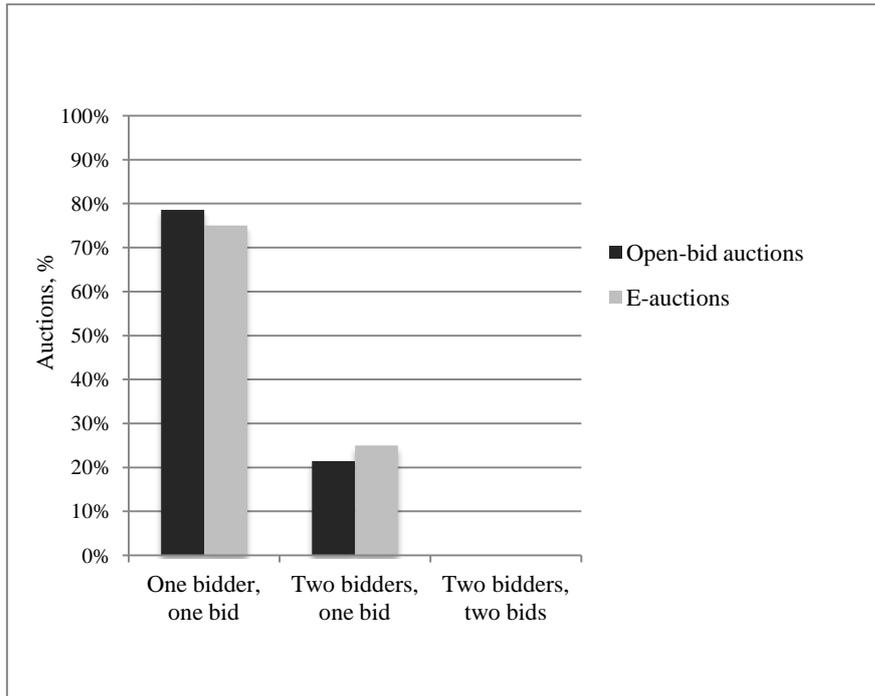
Third, procurer Y reduced the duration of the delivery in several times in e-auctions compared to open-bid auctions, if one considers comparable volumes of gasoline. He asked to deliver huge volume of gasoline for five/ seven days that was very hard to make. For instance, in the December 2010 procurer Y organized an open-bid auction for the delivery of 12'700 liters for approximately one month, while in the September 2012 he organized an e-auction for the delivery of 13'205 liters for seven working days. We consider these requirements as manipulation of delivery terms: they were practically impossible to be met because of production constraints of companies.

Table 6
Some contract characteristics: Procurer Y

Variables	Open-bid auctions		Open-bid auctions		E-auctions	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Reserve price, rub.	3523969	7184544	777964.2	206412.2	770629	435786.3
Total volume of gasoline	212625.2	495160.4	37141.67	10354.6	28114.56	17183.14
Types of gasoline	1.43	1.09	1	0	1.58	.67
24 hours delivery	0	0	0	0	0	0
Reserve price/ market price	.986	.023	0.997	.008	1.037	.017
Obs.	13		12		12	

Procurer Y did not set other requirements that may prevent companies from entry: never asked for round-the-clock delivery of gasoline and did not change the reasonable geographical area where gasoline stations should have been located. Surprisingly, the total number of companies insignificantly increased (see Table 7): an extra bidder started to enter e-auctions more frequently than open-bid auctions. After checking the data more thoroughly, we see that extra bidders registered for participation, but did not make bids in any of e-auctions. Therefore entry of companies increased only because of more frequent passive bidding, which might indicate on coordinated behavior of companies.

Table 7
Bidding in auctions: Procurer Y



The form of passive bidding changed: in open-bid auctions fake companies registered for participation, but did not come to auctions, while in e-auctions passive companies came to auctions, but did not make bids. These passive bidders simulated competition with the preferred bidder, which started to make higher discount in e-auctions compared to open-bid auctions. However one may notice that this discounts were very small (approximately 0.13% on the average and 0.5%-1.1% in each e-auction with passive bidding) and did not cover the increase in reserve prices. As the result, relative prices increased in e-auctions.

Table 8
Entry and prices: Procurer Y

Variables	Open-bid auctions		E-auctions	
	Mean	Std. Dev.	Mean	Std. Dev.
Total bidders	1.21	.43	1.5	.45
Bidders that made bids	1	0	1	0
Passive bidding	.21	.43	.25	.45
Discount	0	0	.0013	.0007
Relative price	.986	.023	1.035	.016
Obs.	13		12	

This case study illustrates both main assumption and obtained results of our theoretical model. First, in Russia public procurers can set strict contract terms, e.g. delivery terms, in order to prevent entry of some companies in public procurement auctions. This behavior results in

lower entry of companies and higher contract prices in practice. We define it as manipulation of contract terms and incorporate into the model of auctions with selective entry. Second, decrease in entry cost after e-procurement reform led to different changes depending on the behavior of public procurers. In case of the procurer X the reform encouraged higher entry of companies and decreased procurement prices, while in case of the procurer Y it had no effect on entry and increased procurement prices. This corresponds to the main result of the model: lower entry cost reduces procurement price paid by the benevolent procurer and may have opposite effect on procurement price paid by the corrupt procurer.

7. Conclusion and discussion

In this paper we present the model of favoritism in procurement auctions by adapting the model of endogenous entry (Samuelson, 1985) to potentially corrupt environment. Unlike Samuelson (1985) and his other followers, we treat a public procurer as a separate player, which can manipulate contract terms in favor of one company and restrict the entry of others. If the procurer is corrupt, he may propose this company to make a contract at the reserve price in exchange for a bribe, so to become his favorite company. If the procurer is benevolent, he could manipulate contract terms to increase the probability of the purchase.

Our main contribution to the economic literature is that the entry cost has different effect on contract prices depending on whether the procurer is benevolent or corrupt¹⁷. When the procurer is benevolent, lower the entry cost decrease the contract price, which corresponds to the result of Samuelson (1985). However when the procurer is potentially corrupt, lower entry cost may provoke favoritism or keep the same incentives for it. In the former case the contract price increases to the level of the reserve price, while in the latter case it stays equal to the reserve price. The reason for this unexpected change is that the bribe that the favorite company can give to the procurer decreases in the entry cost. So if the entry cost becomes lower than the certain threshold, the favorite company can bribe the corrupt procurer. Then he restricts entry of all other companies by manipulating contract terms and, finally, makes a contract with the favorite company at the reserve price. Hence, a small decrease in the entry cost may provoke favoritism and a huge difference between contract prices paid by the benevolent procurer and the potentially corrupt one.

We propose several extensions of the basic model that are in accordance with this conclusion. In the further research we are going to examine how different forms of distribution of production costs affect obtained results.

¹⁷ Under reasonable conditions: when the reserve price is high enough to make favoritism profitable, the procurer has low or practically no monetary incentives to save governmental money and the reserve price is fixed.

The case study of two public procurers, which organized gasoline auctions in Nizhnii Novgorod, Russia, illustrates this result. In 2011 all Russian procurers were obliged to organize e-auctions instead of traditional auctions. This reform decreased the entry cost of companies, so we exploit the difference in the entry cost to examine the shift in entry and contract prices in auctions organized by two different procurers. We find that two procurers started to act in a different way after the reform. One of them manipulated delivery terms, so lower entry cost did not stimulate entry of newcomers, but made passive bidding less costly and more frequent in e-auctions. The other procurer did not manipulate contract terms and benefited from both higher entry and lower prices in e-auctions.

We strongly believe that one may apply the found theoretical result combined with its empirical illustration to the current procurement practice. Absent or low effect of e-procurement reform may indicate that some rent-seeking incentives were not taken into account, rather than contradict the idea of e-procurement itself. Therefore to encourage competition in potentially corrupt environment the government cannot change only the auction format and should use complex measures including anti-corruption techniques.

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APPENDIX

Appendix 1

Figures 2a-2d

Impact of number of companies and the entry cost on expected utilities of procurer

Figure 2a

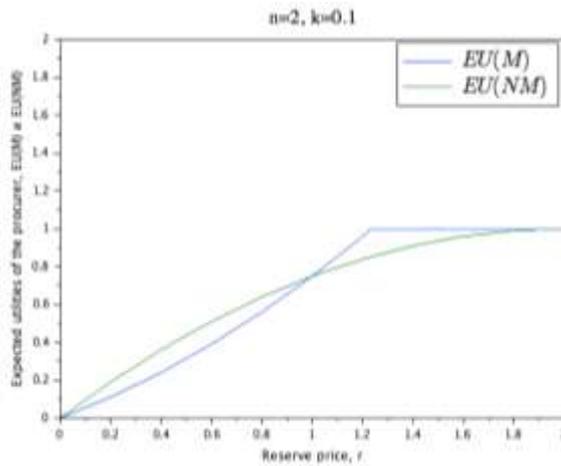
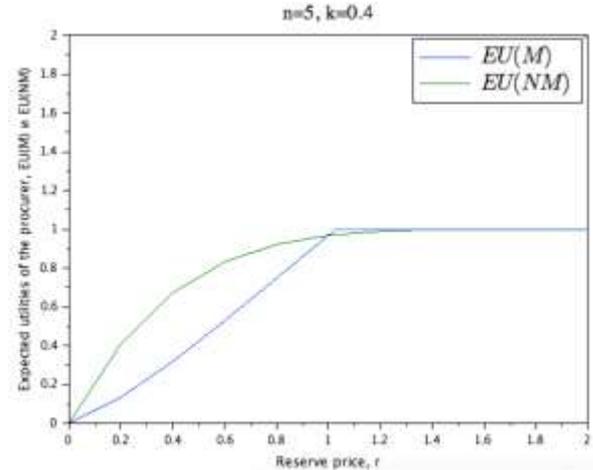


Figure 2b



Appendix 2

Proposition 3

The optimal bribe increases in the reserve price and the number of companies in the market and decreases in the entry cost.

Proof

We take derivatives of the right part of the equation (13) with respect to the entry cost k , the reserve price (r) and the number of bidders (n).

The derivative of the optimal bribe with respect to the entry cost is equal to:

$$\frac{dB^*}{dk} = \frac{dB^*}{dc^*} \cdot \frac{dc^*}{dk}.$$

From to the equation (3) we get that $\frac{dc^*}{dk} = \frac{-1}{[1-c^*]^{n-2} \cdot ((n-1)r - nc^* + 1)}$ and $c^* < r$. Then

taking into account that production costs are lower than 1, $(n-1)r + 1 > nc^*$, hence, $\frac{dc^*}{dk} < 0$, threshold production costs decrease in the entry cost.

From the equation (13) we get the following:

$\frac{dB^*}{dc^*} = (r - c^*)(n-1)[1 - c^*]^{n-2} + [1 - c^*]^{n-1}(1 - c^*)$. Each of the terms is higher than zero, hence, $\frac{dB^*}{dc^*} > 0$, the optimal bribe increases in threshold production costs.

Multiplying two derivatives we get that $\frac{dB^*}{dk} < 0$, the optimal bribe decreases in the entry cost. We have proved the first part of the Proposition 3.

The derivative of production costs with respect to the reserve price equals:

$$\frac{dB^*}{dr} = \frac{dB^*}{dc^*} \cdot \frac{dc^*}{dr}.$$

We have shown above that the derivative of the optimal bribe with respect to threshold production costs ($\frac{dB^*}{dc^*}$) is strictly positive. Then we write the equation (3) in this way:

$$r = c^* + \frac{k}{[1-c^*]^{n-1}}.$$

and get the the derivative of threshold production costs with respect to the reserve price:

$$\frac{dc^*}{dr} = \frac{[1-c^*]^{n-1}}{[1-c^*]^{n-1}+k(n-1)}.$$

As both the numerator and the denominator are positive, $\frac{dc^*}{dr} > 0$, threshold production costs depend positively on the reserve price. Multiplying two derivatives we get that $\frac{dB^*}{dr} > 0$, the optimal bribe increase in the reserve price. We have proved the second part of the Proposition 3.

The derivative of production costs with respect to the number of bidders equals:

$$\frac{dB^*}{dn} = \frac{dB^*}{dc^*} \cdot \frac{dc^*}{dn}.$$

We have shown above that the derivative of the optimal bribe with respect to threshold production costs ($\frac{dB^*}{dc^*}$) is strictly positive. Then we write equation (3) in this way:

$$\frac{k}{(r-c^*)} = [1-c^*]^{n-1},$$

$$\log_{1-c^*} \frac{k}{(r-c^*)} = n-1,$$

and taking the opposite derivative get the equation for the derivative of threshold production costs with respect to the number of bidders:

$$\frac{dc^*}{dn} = -\ln(1-c^*) \cdot (r-c^*).$$

Threshold production costs c^* are lower than 1, $\ln(1-c^*) < 0$, hence, $\frac{dc^*}{dn} > 0$.

Multiplying two derivatives we get that $\frac{dB^*}{dn} > 0$, the optimal bribe increase in the number of bidders. ■

Appendix 3

Proposition 4

For each intermediate value of the reserve price r there is a unique value of the entry cost \tilde{k} when the optimal bribe equals zero:

$$B^*(\tilde{k}) = \frac{1}{3}(c^*(\tilde{k}))^3 - \frac{3}{2}(c^*(\tilde{k}))^2 + (r+1)c^*(\tilde{k}) - \frac{1}{2} = 0. \quad (14)$$

If the entry cost are below this level, company 1 is ready to give a positive bribe and the corrupt deal may take place. If the entry cost exceed this level, company 1 is not ready to give a positive bribe and the corrupt deal never takes place.

Proof

We should prove that the equation (14) has one real root $c^*(\tilde{k})$, when r satisfies the inequality (11). We solve the cubic equation using trigonometric Vieta's formula and find that the real root is unique. This root satisfies all previously made assumptions and corresponds to the adequate level of the entry cost \tilde{k} . ■

Appendix 4

Proposition 5

The expected utilities $EU(M; B)$, $EU(NM; 0)$ decrease in the entry cost. The higher the entry cost are, the higher the contract price the procurer pays in the auction without manipulation and the lower the optimal bribe is in case of manipulation.

Proof

The expected utilities $EU(M; B)$ and $EU(NM; 0)$ equal, respectively:

$$EU(M; B) = 1 + B^* - \beta r \quad (15.1)$$

$$EU(NM; 0) = 1 - [1 - c^*]^n - 2n\beta \int_0^{c^*} c[1 - c^*]^{n-1} dc - \beta n c^* k \quad (15.2)$$

First we consider the expected utility $EU(M; B)$. The reserve price and the financial source β are exogenous; hence, the expected utility of the corrupt procurer in case of favoritism depends on the entry cost in the same way, as the optimal bribe:

$$\frac{dB^*}{dk} = \frac{dEU(M; B^*)}{dk}.$$

From the Proposition 3, $\frac{dB^*}{dk} < 0$. Hence, the expected utility of the corrupt procurer in case of favoritism decreases in the entry cost:

$$\frac{dEU(M; B^*)}{dk} < 0.$$

We have proved the first part of the Proposition 4.

Second we consider the expected utility $EU(NM; 0)$ and re-write it as follows:

$$\frac{dEU(NM; 0)}{dk} = \frac{dEU(NM; 0)}{dc^*} \cdot \frac{dc^*}{dk}.$$

The Proof to the Proposition 3 shows that $\frac{dc^*}{dk} < 0$, threshold production costs decrease in the entry cost. Then we substitute the size of the entry cost from the equation (3) to the equation (15) and take derivative of this with respect to threshold production costs:

$$EU(NM; 0) = 1 - [1 - c^*]^{n-1}(1 - c^* + \beta n r c^* - \beta n (c^*)^2) - 2\beta n \int_0^{c^*} c[1 - c]^{n-1} dc,$$

$$\begin{aligned} \frac{dEU(NM; 0)}{dc^*} &= (n - 1)[1 - c^*]^{n-2}(1 - c^* + \beta n r c^* - \beta n (c^*)^2) - \\ &- [1 - c^*]^{n-1}(-1 + \beta n r - 2\beta n c^*) - 2\beta n c^*[1 - c^*]^{n-1}. \end{aligned}$$

Simplifying this expression, we obtain:

$$\frac{dEU(NM; 0)}{dc^*} = n[1 - c^*]^{n-2}[(1 - c^*)(1 - \beta r) + \beta(n - 1)c^*(r - c^*)]$$

According to the set-up, $n > 1$, $1 > c^*$, $r > c^*$, $\beta r < 1$, hence, the expected utility $EU(NM; 0)$ decreases in threshold production costs:

$$\frac{dEU(NM; 0)}{dk} < 0.$$

As the result, the expected utility of the honest procurer in the absence of manipulation decreases in the entry cost. ■

Appendix 5

Proposition 7

If the procurer manages the public money, the difference between procurer's utilities is zero or negative, $\Delta U \leq 0$. The procurer never benefits from favoritism, and favoritism does not arise.

Proof

We can prove it by addressing two statements. First, the equation (16) has one real root $c(\hat{k})$, when $n = 2$ and $\beta = 1$. This root satisfies all previously made assumptions and corresponds to the adequate level of threshold the entry cost \hat{k} . As in the Appendix 3, we solve the cubic equation using trigonometric Vieta's formula:

$$c(\hat{k}) = -2\sqrt{\left|r^2 - \frac{9}{4}r + \frac{11}{36}\right|} sh(\phi) + \frac{2}{3}r + \frac{1}{6},$$

$$\text{where } \phi = \frac{1}{3} \text{Arch} \left(\frac{\left| \frac{8}{27}r^3 + \frac{7}{9}r^2 - \frac{23}{36}r + \frac{125}{206} \right|}{\sqrt{\left| r^2 - \frac{9}{4}r + \frac{11^3}{36} \right|}} \right).$$

Second, $\Delta U < 0$. When the entry cost does not equal the threshold, $k \neq \hat{k}$, $dU(c(k)) < 0$, the difference between procurer's utilities is negative. ■