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Where has all the education gone? Nowhere, but too much

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Abstract: *Lant Pritchett (2001) asked a famous question, "Where has all the education gone?" bringing the lack of correlation between the growth of measured education and the growth of income in developing countries to broad attention. This finding confirms that after WWII the human capital-output ratios tend to be higher in less developed countries than those in developed countries. I explain this pattern using a dynamic general equilibrium model which explicitly considers that workers with different types have different costs when choosing schooling years and employers are unable to directly observe workers' types, and find that simulation results with public subsidies to schooling could well mimic the features of data. At last, I make a speculative but reasoned conjecture about the schooling years-output relation in 2040.*

I construct a dynamic general equilibrium model for quantitative analysis of measured schooling years and economic growth. This model explicitly takes it into account that workers with different types have different costs when determining schooling years and employers are unable to directly observe workers' types. It can be calibrated to match relevant evidence of education and economic growth after WWII, and generate simulation results under various hypothetical circumstances, such as separating equilibrium, pooling equilibrium, and hybrid equilibrium. The analysis shows that, with only one shock, public subsidies to schooling, the model could well mimic the features of data.

Four circumstances of the model are treated. They are the symmetric information case as the baseline, separating equilibrium, pooling equilibrium, and hybrid equilibrium. Simulation results show that, among these four cases, the simplest model to well mimic observed data is pooling equilibrium with only one shock, subsidies to schooling. Hybrid equilibrium could also mimic data and its settings are more realistic, yet it needs an additional exogenous shock, a threshold separating two educational pools. The underlying intuition of simulations is that over-education due to subsidies to schooling takes scarce resources which should have formed physical capital and obstructs economic growth, and that pooling different types in terms of schooling restrains educational achievements even when education is heavily subsidized. This could be especially true in developing countries.

One motivation of writing this model is to better understand the role of education in growth, which has been long debated since Pritchett (2001) claimed that "on average, education contributed much less to growth than would have been expected in the standard augmented Solow model." In a standard augmented Solow model, both human and physical capital, as factors of production, are generally considered equally important in aggregate production function. And thus they should have had similar effects on output and growth. After finding human capital's role in growth is not as important as physical capital, Pritchett (2001) naturally asked, "Where has all the education gone?"[1]

[1] Actually, Benhabib and Spiegel (1994) found what Pritchett (2001) claimed even earlier, i.e., in the long run, economic growth correlates with the change of education during the same period negatively, and statistically insignificant. However, Benhabib and Spiegel (1994) directly take imputed average schooling years as stock of "human capital", which is replaced by more sophisticated and micro-based approach now.

Later many economists contributed to this issue. For example, Temple (2001) generalized Pritchett's regressions by assuming different specifications for human capital, and did not find significant effects of education on growth. However, Krueger and Lindahl (2001) argued that measurement error in schooling years is the key reason why changes in average schooling years could not explain changes in income per capita from 1960 to nowadays. Furthermore, Cohen and Soto (2007) re-constructed a new data set for schooling years claiming to reduce measurement error present in Barro and Lee's (2001), and found significance in both conventional OLS regressions and more sophisticated panel data regressions.

As a response to the opposite point of view that changes in education could significantly explain growth, Pritchett (2006) listed a number of features of the data on output and schooling across countries. First, historical growth rates are stable in contrast to the fact that schooling levels are trending. Second, output levels are diverging in contrast to the fact that schooling levels are converging. Third, in developing countries growth rates collapse in contrast to the fact that schooling levels are rising. Fourth, growth rates are volatile in contrast to the fact that both schooling levels and growth of schooling are stable. What the four contrasts raised by Pritchett (2006) imply is that, unlike physical capital, human capital-output ratios tend to be higher in less developed countries than those in developed countries.

As far as I can see, this stylized fact is not in conflict with what has been found in Cohen and Soto (2007). Education could have played a positive role in economic growth, while poor countries tended to invest too much in human capital and encumbered their economic performance. And in order to further understand these phenomena, this paper aims to build a dynamic general equilibrium model, which is not only able to account for big facts of growth, but also in which the behavior of individual agents underpins the observed aggregate patterns. In this model three relevant facts are considered: governments subsidize education massively and universally in the world, heterogeneous agents face different incentives and constraints when deciding how many years of schooling they would take, and employers are unable to observe workers' types directly. And this paper contributes to the literature by nesting the above characteristics in a dynamic general equilibrium model, and solving the model.

Lastly, based on the model I make a foolhardy prediction on the relationship between average schooling years and per capita income in 2040. If the behavior of governments in terms of subsidizing education in the world would not change systematically, the schooling years-income relation in 2040 will be unchanged relative to that relation in 2010. This prediction is actually inconsistent with extrapolating the existing pattern of the relationship, which is changing over time. My conjecture could be well proved wrong as time passed. However, I do hope this action could bring the issue of better understanding the role of education in growth to the attention of broader audience.

The rest of the paper is organized as follows. In Section 1 I document that physical capital-output ratios are almost a constant both across countries and over time, but human capital-output ratios are not. In Section 2, I explain why I choose the specifications presented in the paper according to available facts and knowledge related to education and income after WWII.

In Section 3 I lay out the model. In Section 4 I calibrate the model and explore whether the mechanisms adopted in the model are capable of mimicking patterns found in Section 1. And in Section 5 I make the conjecture as concluding remarks.

1. Some Facts

The two plots in Figure 1 show us the correlations between relative income and the relative levels of two capital stocks: physical capital and human capital. The horizontal axes are income per capita relative to that of the United States. So if a number reads 0.5, then it means per capita income in a country is 50% of the U.S. income per capita. Similarly, the vertical axis in the left plot is index of human capital per capita relative to that of the U.S. and the vertical axis in the right plot is physical capital per capita relative to that of the U.S. All these variables come from Penn World Table 8.0 (PWT 8.0, Feenstra, Inklaar and Timmer, 2013). [2] According to the population sizes from large to small, I choose 41 economies from PWT 8.0 with available data.[3] The earliest year in PWT 8.0 is 1950, and I plot the correlations between relative income per capita and relative capital stocks per capita every thirty years, i.e., in 1950, 1980, and 2010.[4] To make these plots easier to read, scatters in 1950 are in red squares, 1980 blue diamonds, and 2010 black circles. I also plot the linear fitted curves for these three years to show the patterns more clearly.

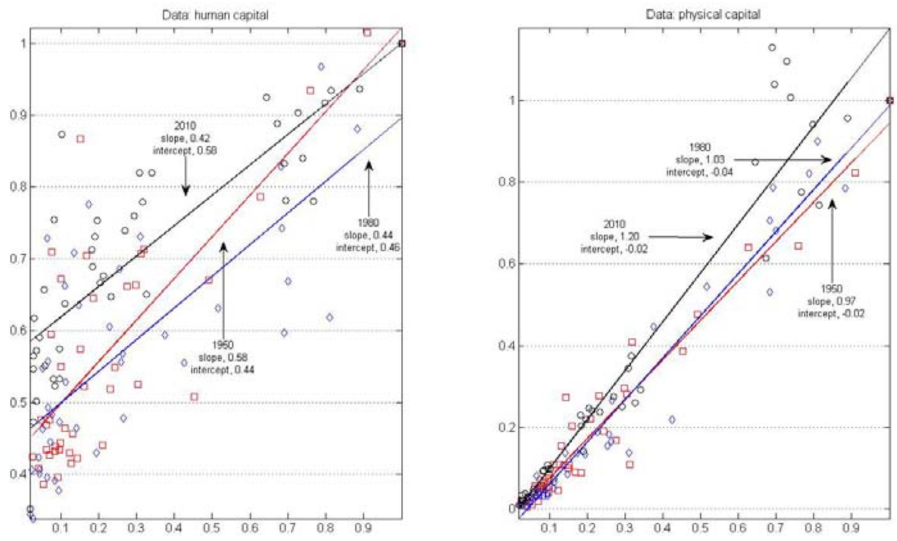

Figure 1: Relative Income versus Relative Capital Stocks

[2] Relative per capita human capital index is based on the "hc" variable in PWT 8.0. Income per capita is calculated by "rgdpe" and "pop" variables. And physical capital is calculated by "ck" and "pop". Physical capital can also be calculated by "rk" and "pop", yet both create similar patterns.

[3] They are China, India, US, Indonesia, Brazil, Pakistan, Bangladesh, Japan, Mexico, Philippines, Egypt, Germany, Iran, Turkey, Thailand, France, UK, Italy, South Africa, Republic of Korea, Colombia, Spain, Tanzania, Argentina, Kenya, Canada, Morocco, Uganda, Venezuela, Peru, Nepal, Malaysia, Ghana, Syria, Australia, Taiwan, Sri Lanka, Mozambique, Cote d'Ivoire, Cameroon, and Chile.

[4] The earliest years for a number of economies are not 1950, but close to 1950. The initial year for these economies are 1951 (Taiwan, Chile), 1952 (China), 1953 (Republic of Korea), 1955 (Iran, Malaysia, Ghana), 1959 (Bangladesh), 1960 (Indonesia, Tanzania, Nepal, Syria, Romania, Mozambique, Cote d'Ivoire, Cameroon).

Index of human capital per capita is based on schooling years (Barro and Lee, 2013) and returns to education (Psacharopoulos, 1994). Basically the method assumes that human capital is proportional to the wage rate, and can be deduced from the robust relation between schooling and wage rate. This method of estimating human capital is now widely used in the literature (e.g., Klenow and Rodriguez-Clare, 1997; Hall and Jones, 1999). Figure 2 compares human capital index and average schooling years in the same space as in Figure 1. And these two variables show similar pattern both across economies and over time.

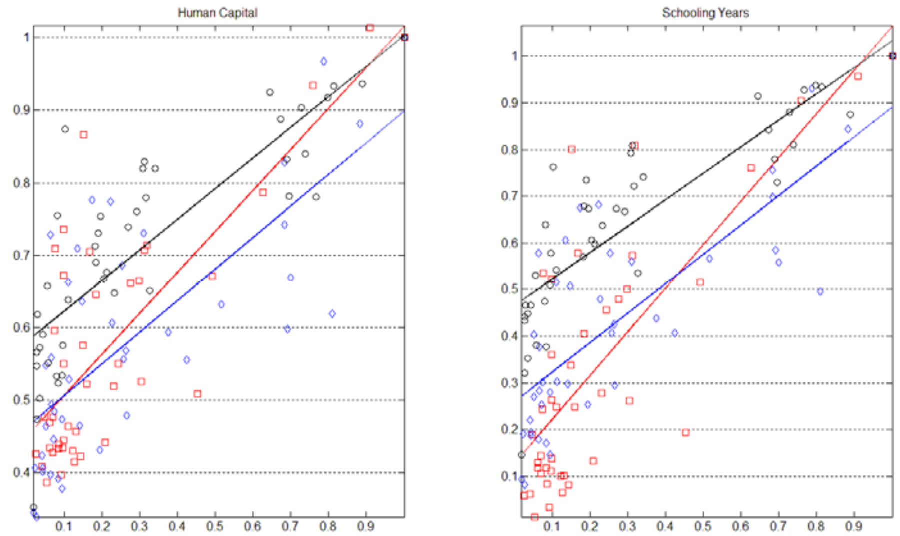

Figure 2: Human Capital Index and Average Schooling Years

Figure 1 clearly shows that physical capital-output ratios are almost a constant both across countries and over time, because the three fitted curves for the physical capital-income relation are all through the origin and with a slope close to one. In specific, the slope of the fitted curve in 1950 is 0.97 and the intercept is -0.02; the slope of the fitted curve in 1980 is 1.03 and the intercept is -0.04; the slope of the fitted curve in 2010 is 1.20 and the intercept is -0.02. In 2010, the physical capital-output ratios become higher for richer economies than before suggesting a remarkable change beyond the scope of this paper.

Such stable pattern does not hold for human capital. In the left plot of Figure 1 we can see that the slope of the fitted curve in 1950 is 0.58 and the intercept is 0.44; the slope of the fitted curve in 1980 is 0.44 and the intercept is 0.46; the slope of the fitted curve in the 2010 is 0.42 and the intercept is 0.58. Hence, for poorer economies human capital-output ratios are higher than those in richer economies in 1950, and it becomes continually higher in 1980 and 2010. This different pattern between physical and human capital is what the paper attempts to account for.

2. Related Literature for Modeling

When it comes to modeling, at least in theory, available modeling setups are infinitely many. Hence, the very principle about modeling of the paper is that stylized facts and solid knowledge should restrict modeling options, though it is subjective to tell what facts or knowledge are substantial and deserve priority.

The first choice I made about modeling is that the model should be tractable, consistent with big growth facts, and flexible enough to incorporate heterogeneous agents, informational asymmetry, and subsidies to schooling. The "impure altruism" model proposed by Andreoni (1989) well satisfies these requirements. It has an overlapping generations structure, and has some clear parallels to the canonical overlapping generations model, especially in that it leads to equilibrium dynamics very similar to those of the Solow growth model. Because of its advantages, now the "impure altruism" model is widely used for studying economic dynamics involving imperfect capital markets in the growth and development literature (e.g., Aghion and Bolton, 1997).

The second choice about modeling is that human capital is a productive factor of production, but not purely a signaling mechanism about workers' types. The empirical evidence supporting this statement is Mincer regressions, named after Jacob Mincer (1974). It specifies a regression equation, where dependent variable is the logarithm of wages, while independent variables are schooling years, experience, and squares of experience. Mincer equation has been estimated for a large number of economies and time periods, while in most cases the estimates are quite robust from 0.05 to 0.15 (Psacharopoulos, 1973, 1985, 1994; Psacharopoulos and Patrinos, 2002). As Thomas Lemieux (2006) says, Mincer equation has been "one of the most widely used models in empirical economics". It is suggestive to say that the productive role played by schooling in the production process underpins the robust, statistically significant and positive estimates of Mincer regressions.

The third choice about modeling is still about the role played by measured education in production process –schooling also sends signals about heterogeneous types of workers when employers are unable to directly observe workers' types, but able to directly observe workers' years of schooling. Although the supporting empirical evidence to this statement is not as much as Mincer regressions, Tyler, Murnane, and Willet (2000) still provide a convincing piece of such evidence showing that signaling effects at least exist for a specific diploma (General Educational Development), in a specific time period (postwar), and in a specific economy (the United States).

The fourth set of choices about modeling is about two exogenous shocks. First I choose government subsidies to schooling as the wedge into investing human capital. Intuitively, with educational subsidies, human capital investment would move upward compared with the level without such subsidies, and physical capital and economic performance would change accordingly. Nowadays and in history governments subsidize education in almost all countries in the world. Public spending on education is one of the largest government outlays. And in most countries, the government is the main provider of education services, even though these services can, in principle, be provided privately. Furthermore, public sources of funding represent a significant part of income for private schools, when private schools are available (Glomm, Ravikumar, and Schiopu, 2011).

Another noticeable characteristic of modern education system is laws mandating compulsory education, which require all children of both genders to attend school up to a certain age. Prussia was the first to enact modern compulsory education legislation (Soysal and Strang, 1989). Nowadays some kind of education is compulsory to all people in most countries, but different countries vary in how many years or grades of education they require. Apparently, the workforce with completed and above compulsory education serves in different industries, vocations, or positions compared to the rest of workforce. If education did send signals about workers' types, then this fact characterizes, at least part of, information structure in the education signaling game. And as a result, human capital investment and economic performance would be affected. In the model I use compulsory grades as the exogenous thresholds to separate educational pools in the hybrid equilibrium setup. Figure 3 shows the relation of private educational expenditure share and above-compulsory education labor force share across countries. The plot shows that these two variables are largely independent.

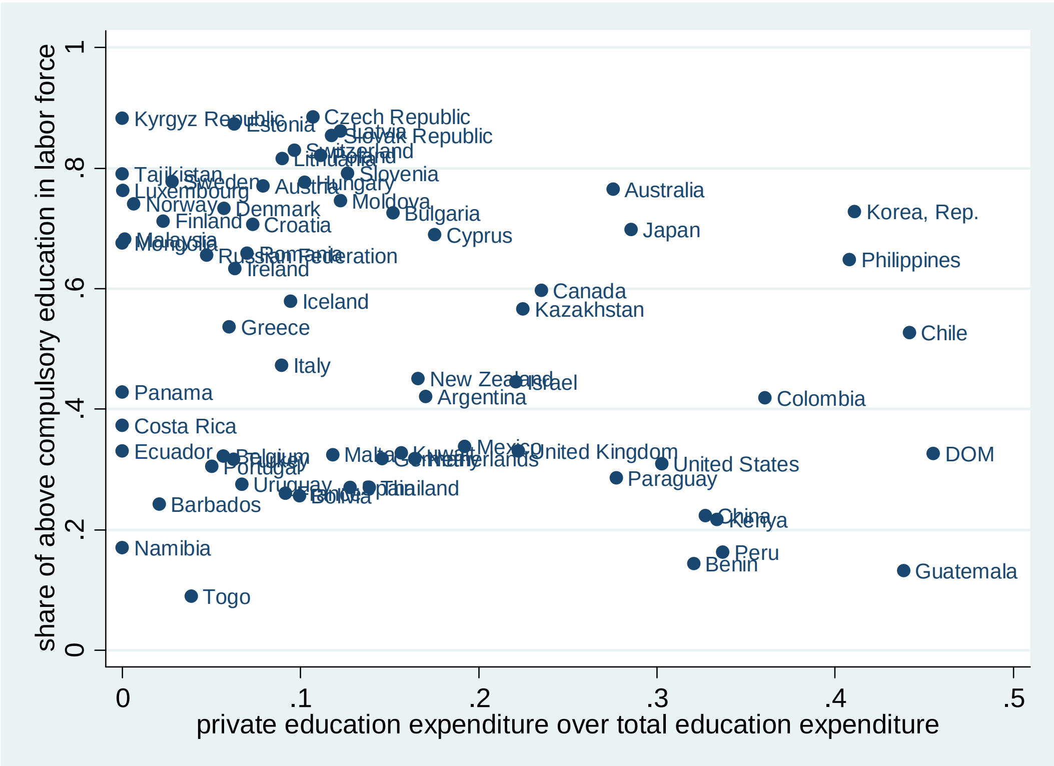

Figure 3: Private Educational Expenditure Share versus Above Compulsory Education Labor Force Share

Source: World Bank, EdStats, core indicators.

These modeling choices underpin the assumptions and specifications of the model presented in the next section, like impure altruism model, heterogeneous workers, informational asymmetry on types between employers and workers, government subsidies to education and so on.

3. Model

Because of its tractability, flexibility, and consistency with growth facts, the "impure altruism" preferences are adopted in the paper. This specification assumes that parents derive utility from their bequest to their offspring, rather than from the utility or the consumption of their offspring.

Assume that the economy is populated by a continuum of agents of measure one. Every agent lives for two periods, childhood and adulthood. In adulthood, the second period of its life, each agent begets an offspring, consumes, invests, works, bequeaths, and then quits the economy. Thus population is constant. For simplicity, assume that during childhood, agents do not consume (or their consumption is incorporated in the parent's consumption). Also assume that there is no labor-leisure tradeoff for any agent, and thus all adults supply their one unit of labor inelastically.

Assume the utility function of adult i at time t in adulthood is,

$$u(c_{it}, b_{it}) = c_{it}^{\eta} b_{it}^{1-\eta}$$

where c_{it} is the consumption of the adult agent i at time t, and b_{it} is the bequest to its offspring. The function form is Cobb-Douglas, so the share of consumption or bequest to total income is constant. Simultaneously the offspring is in its childhood. In the next period, the offspring grows up with the bequest, invests part of the bequests as assets, works, makes consumption and bequest decisions, and begets its own offspring. Here assume that capital depreciation rate is one.

Then the utility maximization problem becomes,

$$\max c_{it}^{\eta} b_{it}^{1-\eta}$$

subjective to,

$$c_{it} + b_{it} \leq m_{it} = w(h_{it}) + R_t a_{it}$$

where m_{it} is the total income of adult i at time t, which consists of labor income $w(h_{it})$, a function of human capital h_{it}, and capital income, a product of assets a_{it} and asset return R_t.

In childhood, offspring invests part of the bequest to accumulate human capital used in the next period, and the cost function of investing human capital is as follows,

$$(1) x_{it-1} = \frac{Zh_{it}^{1+\phi}}{\theta_i}$$

where x_{it-1} is the human capital investment in childhood, which depends on the human capital available in adulthood h_{it}, and the type of offspring θ_i. Z is parameter which reflects cost efficiency of investing human capital, and ϕ is another positive parameter assuring that marginal cost of investing human capital is increasing.

Then the income maximization problem becomes,

$$\max w(h_{it}) + R_t a_{it}$$

subjective to,

$$(2) a_{it} + (1-\tau) x_{it-1} + T_{it-1} \leq b_{it-1}$$

and equation (1). In equation (2), τ is government subsidies (taxes) to education if positive (negative), and T_{it-1} is lump sum taxes (or transfers) to assure that for any time period government budget is in balance, $T_{it} = \tau x_{it}$.

Assume that the production function is Cobb-Douglas,

$$(3) y_{it} = \theta_i k_{it}^{\alpha} h_{it}^{1-\alpha}$$

where y_{it} is the output produced by adult i at time t, k_{it} is the physical capital equipped to i, and α is the conventional income share parameter. Notice that the type determining human capital investment θ_i also determines production efficiency. And a more cost-efficient worker in investing in human capital is also more efficient in producing output.

Then the profit maximization problem faced by the representative firm is,

$$\max \int_0^1 [y_{it} - w(h_{it}) - R_t k_{it}] di$$

To close the model, a market clearing condition is needed,

$$(4) \int_0^1 k_{it+1} di = \int_0^1 a_{it+1} di$$

which means that total assets invested by households should be equal to total physical capital used by firms.

Now only information set in the model has not been defined yet. Assume that workers know their types, but firms may or may not observe workers' types. If both workers and firms know types, then the model turns out to be a conventional growth model, where all endogenous variables should satisfy market clearing condition, and equilibrium conditions implied by utility maximization problem, income maximization problem, and profit maximization problem. If firms are unable to observe types of workers, then the equilibrium of the model must be a perfect Bayesian equilibrium (PBE), and human capital levels send signals about workers' types.

Assume the timing of the signaling game as follow,

(1) Workers find their types.
(2) Firms offer a wage table depending on observable human capital levels.
(3) Workers choose human capital investment levels, and would accept associated wages.
(4) Firms equip workers with physical capital.
(5) Workers produce output.

Generally speaking, there are three types of equilibrium in a signaling game: separating equilibrium, pooling equilibrium and hybrid equilibrium. No matter what case it is, an equilibrium must satisfy the following definition of a perfect Bayesian equilibrium.

(1) Given the wage table, workers' choices are optimal.
(2) Firms' subjective distribution of workers' types is formed according to the Bayes rule.
(3) Given subjective distribution of workers' types and workers' choices, firms' choices are optimal.

Typically, in a signaling game, a special case of dynamic games of incomplete information, the standard way to derive first-order conditions is backward induction. This requires that in the first step workers' choices satisfy both incentive compatibility and participation constraint. Notice that agents could only choose human capital investment, but not their types. Incentive compatibility means that, given private information, truthfully revealing types is at least as good as mimicking other agents' human capital levels for any agent, and participation constraint requires every agent being active. According to agents' optimal choices, firms then maximize profit in the second step.

In order to fully explore the implications of the above setup, I derive the first-order conditions of four circumstances in the model: symmetric information (baseline), separating equilibrium, pooling equilibrium, and hybrid equilibrium (two pools).

Baseline

Assume that both firms and workers know workers' types. Then the equilibrium conditions of the utility maximization problem are, $\forall i$

$$(5)\, c_{it} = \eta m_{it}$$

$$(6)\, b_{it} = (1-\eta) m_{it}$$

$$(7)\, m_{it} = w_{it} h_{it} + R_t a_{it}$$

And the equilibrium conditions of the income maximization problem are, $\forall i$

$$(8)\, \frac{dw_{it}}{dh_{it}} = \frac{R_t (1-\tau)(1+\phi) Z h_{it}^{\phi}}{\theta_i}$$

$$b_{i0} = b_0$$

And the equilibrium conditions of the profit maximization problem are, $\forall i$

$$(9)\, R_t = \alpha \theta_i k_{it}^{\alpha-1} h_{it}^{1-\alpha}$$

$$(10)\, \frac{dw_{it}}{dh_{it}} = (1-\alpha) \theta_i k_{it}^{\alpha} h_{it}^{-\alpha}$$

Equation (1)-(10) fully characterize the model equilibrium when no informational asymmetry exists. It is worth noticing that combining equation (8) and (10) implies for any type its marginal product of human capital should be equal to its marginal cost of investing human capital. Obviously when government subsidies are absent, equilibrium is also Pareto optimal, while education is in excess when subsides are present.

Now consider the cases that only workers know their types, but firms do not. It is useful to consider separately three types of equilibrium that might arise: separating equilibrium, in which each type chooses a unique human capital level, pooling equilibrium, in which all types choose the same human capital levels, and a simple hybrid equilibrium, in which, I assume, only two human capital levels are feasible, and some types choose a lower level, while others choose a higher one.

Before showing the equilibrium conditions, we first need derive an important result.

Lemma: Incentive compatibility alone requires $\frac{dh}{d\theta} > 0$.

Proof: Please see appendix.

The above lemma simply means that in any equilibrium which is incentive compatible higher types tend to choose higher levels of human capital.

Separating Equilibrium

First let us consider separating equilibrium. Solving equilibrium conditions involves backward induction. Given the wage rates offered by firms and rate of return to physical capital, children make investment decisions to maximize their income. Remember that workers can only choose their human capital levels but not their types. Define a worker's indirect utility function, $U(\theta) = w - R(1-\tau)x$. Then a similar first order condition to (8) arises, $\forall\theta$

$$\frac{dw}{dh}[h(\theta)] - R(1-\tau)\frac{\partial x}{\partial h}[h(\theta),\theta] = 0$$

And the associated second order condition is,

$$\frac{d^2w}{dh^2}[h(\theta)] - R(1-\tau)\frac{\partial^2 x}{\partial h^2}[h(\theta),\theta] \le 0$$

Differentiating the first order condition by θ,

$$\left[\frac{d^2w}{dh^2}[h(\theta)] - R(1-\tau)\frac{\partial^2 x}{\partial h^2}[h(\theta),\theta]\right]\frac{dh}{d\theta} - R(1-\tau)\frac{\partial^2 x}{\partial h\,\partial\theta}(h(\theta),\theta) = 0$$

Remember that $\frac{dh}{d\theta} \ge 0$, shown by the Lemma. Then the second order condition can be written as follows,

$$\frac{\partial^2 x}{\partial h\,\partial\theta}[h(\theta),\theta] \le 0$$

If the above condition is a strict inequality, then it is the Spence-Mirrlees (single crossing) condition in the principal-agent literature. Economically, it simply says that a more efficient type is also more efficient at the margin cost of human capital investment.

Notice that firms take the observed investment behavior of workers into account and form subjective judgment about workers' types when maximizing their profits. The profit maximization problem faced by the representative firm is, $\forall t$

$$\max\int_{\underline{\theta}}^{\overline{\theta}}[y(\theta_i) - U_i - R(1-\tau)x - Rk_i]dF(\theta_i)$$

subject to the first order condition, second order condition, and participation condition.

After some algebraic manipulation, we can derive the first order condition with respect to human capital becomes, $\forall$ i

$$(11)\quad (1-\alpha)\theta_i k_{it}^{\alpha} h_{it}^{-\alpha} = R_t(1-\tau)\left[\frac{\partial x}{\partial h}[h(\theta),\theta] - \frac{1-F(\theta_i)}{f(\theta_i)}\frac{\partial^2 x}{\partial\theta\,\partial h}[h(\theta),\theta]\right]$$

For the detailed derivation, please see the appendix. The intuition behind equation (11) is consistent with concepts and conclusions in information economics. The term after the minus sign inside brackets is the so-called "information rent" in information economics, which is transferred from firms to workers because workers enjoy some informational advantage. Since higher types are more cost efficient in human capital investment, even firms are able to maintain the lowest type of worker at its reserve utility level, higher types could always mimic lower types, and get the information rent, as long as firms insist on all workers being active.

Notice that separating equilibrium differs from the baseline only quantitatively. Equation (11) says that for any type its marginal product of human capital equals its marginal cost plus a positive term, but not changing the structure of equilibrium condition.

Pooling equilibrium

In pooling equilibrium, all types choose the same human capital level. Since the lowest type has the highest cost of human capital investment, among all types its net utility from investing in human capital should be the lowest. If the lowest type satisfies participation condition, then all other types must satisfy participation condition too. Let the participation condition for the lowest type be binding,

$$w_t^p - R_t(1-\tau)x\left[h_t^p, \underline{\theta}\right] = 0$$

where h^p is the universal level of human capital taken by all workers, w^p is the universal labor income received by all workers.

Recall that firms insist on all workers being active in production and take the binding participation constraint into account when maximizing their profits. The profit maximization problem faced by the representative firm is, $\forall t$

$$\max \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta\left[k_t^p\right]^{\alpha}\left[h_t^p\right]^{1-\alpha} - w\left[h_t^p\right] - R_t k_t^p\right] dF(\theta)$$

subject to,

$$w\left[h_t^p\right] = R_t(1-\tau)x\left[h_t^p, \underline{\theta}\right]$$

then the first order conditions are,

$$E[\theta]\alpha\left[k_t^p\right]^{\alpha-1}\left[h_t^p\right]^{1-\alpha} = R_t^p$$

$$(12)\quad E[\theta](1-\alpha)\left[k_t^p\right]^{\alpha}\left[h_t^p\right]^{-\alpha} = R_t^p(1-\tau)(1+\phi)\frac{Z\left[h_t^p\right]^{\phi}}{\underline{\theta}}$$

The above condition describes that in pooling equilibrium the marginal product of human capital averaging over all workers should be equal to the marginal cost of human capital investment of the lowest type. Since the marginal cost of the lowest type is the highest among all types, in pooling equilibrium human capital investment is significantly depressed.

Hybrid equilibrium

Suppose that there are two available educational levels h^1 and h^2, with $h^2 > h^1$. Some workers choose h^1, and the others choose h^2. Remember the Lemma implies that incentive compatibility alone requires higher types tend to choose higher educational levels. Let the associated type threshold be θ^*.

Similar to pooling equilibrium, the binding participation constraint requires,

$$(13)\ w[h^1] = R(1-\tau)x\left[h^1, \underline{\theta}\right]$$

where w^1 is the constant wage rate for all types below the threshold. In addition, the incentive compatibility constraint requires,

$$w[h^2] - R(1-\tau)x[h^2, \theta^*] \geq w[h^1] - R(1-\tau)x[h^1, \theta^*]$$

where w^2 is the constant wage rate for all types above the threshold. The above condition means that for the threshold type choosing higher human capital level is at least as good as choosing lower human capital level. In equilibrium, it is binding only for the threshold type,

$$(14)\ w[h^2] = R(1-\tau)\left[x[h^2, \theta^*] + x\left[h^1, \underline{\theta}\right] - x[h^1, \theta^*]\right]$$

Similar to pooling equilibrium, firms hire all workers and take both the binding participation constraint and the incentive compatibility condition into account when maximizing their profits. The profit maximization problem faced by the representative firm is, $\forall t$

$$\max \int_{\underline{\theta}}^{\theta^*} [\theta_i[k^1]^\alpha[h^1]^{1-\alpha} - w[h^1] - Rk^1]dF(\theta_i) + \int_{\theta^*}^{\overline{\theta}} [\theta_i[k^2]^\alpha[h^2]^{1-\alpha} - w[h^2] - Rk^2]dF(\theta_i)$$

First order conditions with respect to physical capital are,

$$E[\theta^1]\alpha[k_t^1]^{\alpha-1}[h_t^1]^{1-\alpha} = R_t$$
$$E[\theta^2]\alpha[k_t^2]^{\alpha-1}[h_t^2]^{1-\alpha} = R_t$$

Substitute equation (13) and (14) into the maximand,

$$\max E[\theta^1][k^1]^\alpha[h^1]^{1-\alpha} - R(1-\tau)x\left[h^1, \underline{\theta}\right]F(\theta^*) + E[\theta^2][k^2]^\alpha[h^2]^{1-\alpha}$$
$$- R(1-\tau)\left[x[h^2, \theta^*] + x\left[h^1, \underline{\theta}\right] - x[h^1, \theta^*]\right][1 - F(\theta^*)]$$

And first order conditions with respect to human capital are,

$$(15)\ E[\theta^1](1-\alpha)[k^1]^\alpha[h^1]^{-\alpha} = R(1-\tau)\left[\frac{\partial x}{\partial h^1}\left[h^1, \underline{\theta}\right] - \frac{\partial x}{\partial h^1}[h^1, \theta^*][1 - F(\theta^*)]\right]$$

$$(16)\ E[\theta^2](1-\alpha)[k^2]^\alpha[h^2]^{-\alpha} = R(1-\tau)\frac{\partial x}{\partial h^2}[h^2, \theta^*][1 - F(\theta^*)]$$

Equation (15) and (16) characterize the human capital investment behavior in this hybrid equilibrium. Equation (15) is similar to the condition in separating equilibrium, while equation (16) is similar to the condition in pooling equilibrium. This set of equations tell us that not only the wedge like subsidies to schooling could distort resource allocation between physical and human capital, but also specific institutional environment like compulsory education could distort resource allocation within human capital investment.

Compared with the preceding three circumstances, hybrid equilibrium requires an additional exogenous variable, the type threshold, to close the model. Although this characteristic complicates modeling, it is closer to the reality because in reality there exists multiple but limited educational options, which cannot be captured by separating equilibrium or pooling equilibrium.

4. Calibration and Simulation

The above section has worked out four competing theories or models for the relations between capital stocks and income. However, it is not straightforward to see their quantitative implications in terms of explaining the observed income-capital relation by scrutinizing equilibrium conditions. In this section, I turn to the task of quantifying the implications of these four models, and testing their implications against data.

A big difficulty in performing such an exercise is coming up with reasonable measures of relevant policies for such a long time period. Doubtlessly subsidies to schooling were not the only factor determining variations in income and capital stocks. However, given the limited availability of information, what I do is take the government subsidies to schooling from 1998 to 2011 that the World Bank-EdStats collects and assumes the average subsidy intensities during that period apply to the whole postwar period. By doing this exercise I can show what can be learned from a comparison of theories to data. For all these four models, I generate datasets and compare them to Figure1. This is done by producing analogous of Figure1 for the four theories.

Calibration

In order to simulate the theories, we need to choose parameter values. Table 1 reports the calibrated parameter values. All parameter values are chosen to be the same in the four theories. In particular, we choose physical capital share equal to 0.35, human capital investment parameter equal to 8.8828, consumption share equal to 0.6470.

Table 1: Parameter Values

Model components		**Parameter values**	
Production:	$y_{it} = \theta_i k_{it}^{\alpha} h_{it}^{1-\alpha}$	α	0.35
Human capital investment	$x_{it-1} = \frac{Zh_{it}^{1+\phi}}{\theta_i}$	ϕ	8.8828
Preferences	$u_{it} = c_{it}^{\eta} b_{it}^{1-\eta}$	η	0.6470

Physical capital share is an important parameter regularly calibrated in the literature, and 0.35 falls in the range typically used. Human capital investment parameter does not appear often in the literature. Here according to the following equation, I calculate a value of 8.8828 for it.

$$\frac{\text{labor income}}{Y} = \frac{\int_0^1 w_i h_i di}{Y} = \frac{R(1+\phi)\int_0^1 x_i di}{Y} = \frac{R(1+\phi)X}{Y}$$

The above equation specifies the relation between educational expenditure and labor income in steady state of the baseline model. The left hand side of the first equation is the share of labor income in total income, which is equal to $1-\alpha$, while the right hand side of the first equation comes from the definition of labor income. The second equation comes from equation (8). And the last equation uses the definition of total educational expenditure, and its right hand side is proportional to the ratio of total educational expenditure over total income. Taking advantage of the fact that the average ratio of educational expenditure over GDP in the U.S. in history is 7%, the value of human capital investment parameter can be calibrated together with rate of return to physical capital investment.

As for the consumption share, let us rewrite the utility function as,

$$u = \log c + \frac{1-\eta}{\eta} \log b$$

Imagine that bequest determines the welfare of the next generation (or period). Then compare it to a two-period additive utility function,

$$u = \log c + \beta \log c_{+1}$$

where β is the subjective discount factor. Then $\frac{1-\eta}{\eta}$ is analogous to the discount factor. Recall that in the literature a typical annual value of discount factor is 0.98. And the time interval in the model is thirty years. Then,

$$\frac{1-\eta}{\eta} = \beta^{30}$$

The calculated value for consumption share is 0.6470, which implies a saving rate of 35.3%.

For simplicity, assume that types of workers are uniformly distributed on an interval $[\underline{\theta}, \overline{\theta}]$, with an upper bound 1.5, and a lower bound 0.5. The relation between agent i and its type θ is as follows,

$$\theta(i) = (\overline{\theta} - \underline{\theta})i + \underline{\theta}$$

There are two exogenous shocks, governmental subsidies to schooling and threshold type, in the model. From the World Bank-EdStats, I collect both cross-country shares of private spending on education and shares of labor force with above compulsory education for seventy-five countries.[5] According to this observed distribution, I randomly pick fifty times to generate exogenous shocks. For all four theories, assume all economies face only one shock, subsidies to schooling. All other taxes are assumed to be zero. As for hybrid equilibrium one additional institutional characteristic, threshold type, appears. Lastly, generate the initial physical capital stocks evenly from 5% to 100% of the steady state levels.

Simulation Results

Simulation results of the baseline, separating equilibrium, pooling equilibrium, and hybrid equilibrium are summarized in Figure 3-6. All four sets of results should be compared to their analogues in the data, namely, Figure 1.

[5] These countries include Costa Rica, Ecuador, Kyrgyz Republic, Mongolia, Namibia, Panama, Tajikistan, Luxembourg, Malaysia, Norway, Cuba, Barbados, Finland, Sweden, Togo, Russia, Portugal, Guyana, Belgium, Denmark, Belize, Greece, Turkey, Estonia, Ireland, Uruguay, Romania, Croatia, Austria, Italy, Lithuania, France, Iceland, Switzerland, El Salvador, Bolivia, Hungary, Czech Republic, Poland, Slovak Republic, Malta, Belarus, Latvia, Moldova, Slovenia, Spain, Mali, Thailand, Germany, Bulgaria, Kuwait, Netherlands, New Zealand, Argentina, Cyprus, Mexico, Israel, United Kingdom, Kazakhstan, Canada, Australia, Paraguay, Japan, United States, Benin, China, Burkina Faso, Kenya, Peru, Colombia, Philippines, Republic of Korea, Guatemala, Chile, Dominican Republic.

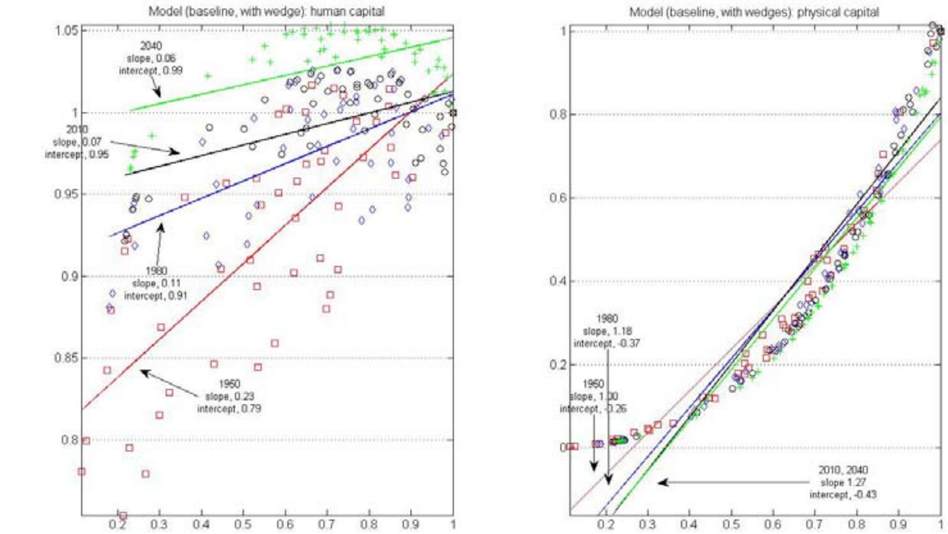

Figure 4: Results for Baseline, Relative Income versus Relative Capital Stocks

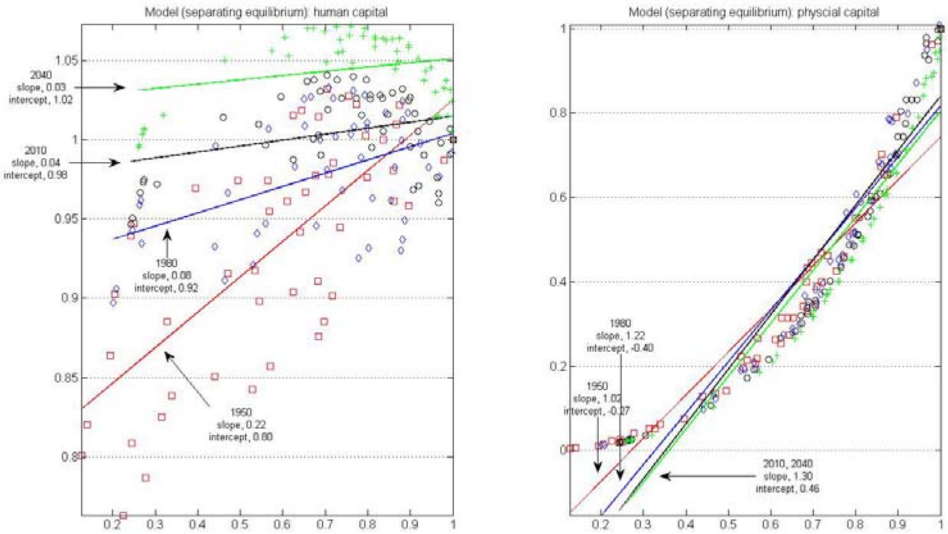

Figure 5: Results for Separating Equilibrium, Relative Income versus Relative Capital Stocks

Figure 4 displays the simulation results of the baseline. Its left plot shows the dynamics of the income-human capital relation, while its right plot shows the income-physical capital relation. In particular, the relations in 1950, 1980, 2010 and 2040 are plotted. Obviously from the right plot the physical capital-output ratios are quite stable over time. However, the pattern for human capital is different. Human capital-output ratios are changing over time, and relatively poor countries accumulate relatively more and more human capital. Furthermore, the cross-sectional correlations between income and human capital in a specific time point are gradually waning over time. This is consistent with the data, yet the scatters generated by model display an inverted-U shape for the upper envelope of scatters. This means that rich but not the richest countries accumulate higher human capital levels than the richest countries. This is inconsistent with the data.

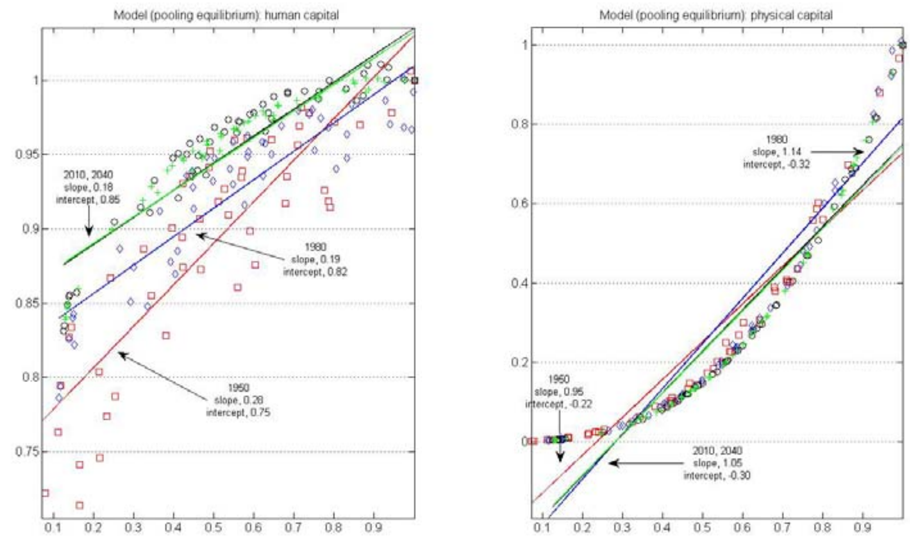

Figure 6: Results for Pooling Equilibrium, Relative Income versus Relative Capital Stocks

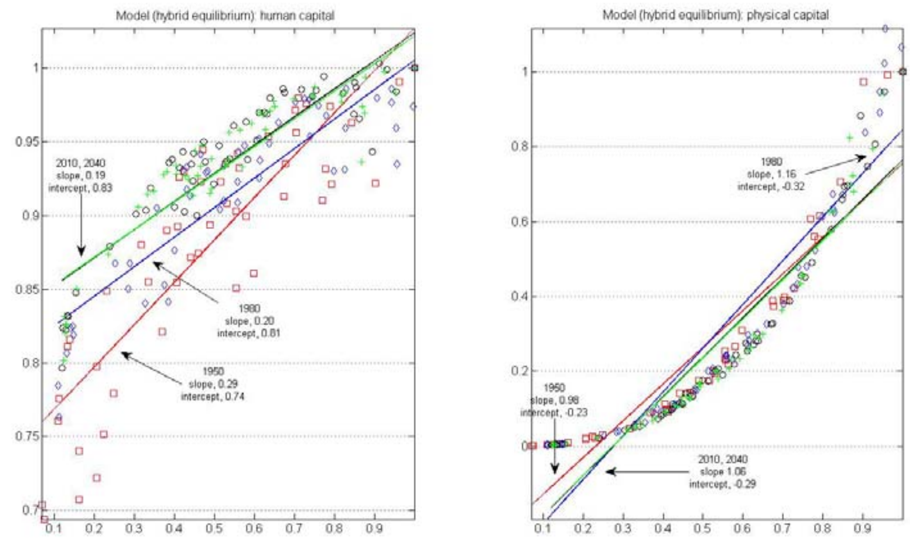

Figure 7: Results for Hybrid Equilibrium, Relative Income versus Relative Capital Stocks

In Figure 5 I display the simulation results in the separating equilibrium case. They are quite similar to the results in the symmetric information case. Physical capital-output ratios are almost a constant over time, and human capital-output ratios are changing over time. Recalling that the equilibrium condition describing human capital investment in separating equilibrium is only changed marginally to that in the baseline, these results are not a surprise.

Figure 6 shows the simulation results of pooling equilibrium. The same as before, the relations between relative income and relative physical capital are stable over time, similar to the above two cases. But compared with the above two cases, the pattern for human capital are closer to the data in two aspects. First, the inverted U shape of the relative income versus relative human capital relation disappears. Or generally speaking, the richest countries also enjoy the highest levels of human capital. Second, the change of fitted curves from 1950 to 1980 is mainly a decrease of the slope, while the change of fitted curves from 1980 to 2010 is mainly an increase of the intercept. This is almost the same as the characteristics in Figure 1.

Figure 7 shows the simulation results of hybrid equilibrium. Qualitatively, its characteristics are the same as those of pooling equilibrium, though hybrid equilibrium is more demanding in modeling.

In summary, all four theories predict the same pattern for physical capital, which suggests physical capital-output ratios are stable over time. As for human capital, compared with the baseline and separating equilibrium, pooling equilibrium and hybrid equilibrium could well mimic the data, though hybrid equilibrium models are more complicated in modeling.

5. Concluding Remarks

In order to answer Lant Pritchett's question: "Where has all the education gone?" and address the issue that human capital-output ratios tend to be higher in less developed countries than those in developed countries, I present a dynamic general equilibrium model with heterogeneous agents, informational asymmetry, and government subsidies to schooling. By this model, I would like to answer Lant Pritchett's question in this way: education did not disappear, but skilled workers do not produce much when they are equipped with insufficient physical capital.

Although putting these characteristics together definitely involves my personal judgment, the paper provides a micro-foundation that could underpin the observed aggregate patterns in terms of income-capital relations during the postwar period. I do not argue that the presented channels are the only explanation to Lant Pritchett's question, yet it is a well established laboratory for exercising some interesting counter-factual experiments and even predicting the future.

One possible prediction is about anticipating the correlation between average schooling years and income per capita in 2040. Figure 2-5 report the fitted curves about the human capital-income relations in 2040. Notice that in pooling and hybrid equilibrium, the human capital-income relation in 2040 is almost unchanged to that in 2010, while in the baseline and separating equilibrium such relations become weaker than that in 2010. Bear in mind that human capital in theory mimics the human capital index in data, which is constructed by average schooling years and the Mincer estimates. If, as suggested by the above section, pooling equilibrium and hybrid equilibrium mimic data better than the baseline and separating equilibrium, then I would predict that the cross-sectional correlation between relative average schooling years and relative per capita income will be unchanged in 2040, compared to the situation in 2010, as long as the Mincer regression estimates and the government behavior of subsiding education would not change systematically.

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Key words: education, informational asymmetry, signaling model, general equilibrium, heterogeneity

JEL Classification: D82, E24, I25, O15, O47

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